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SINGER: A COMPUTER CODE FOR GENERAL ANALYSIS OF TWO-DIMENSIONAL REINFORCED CONCRETE STRUCTURES. VOLUME 1. SOLUTION PROCESS

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Virginia Polytechnic Institute and
State University

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Volume I
Solution Process

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R. J. Melosh
R. M. Barker
A. E. Somers

May 1975

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DEPARTMENT OF CIVIL ENGINEERING
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This report describes the mathematical model and the solution process which form the basis of the computer program SINGER. The function of SINGER is to predict the behavior of plane skeletal reinforced concrete structures in their environments. Of primary interest is the transient nonlinear response including element failures and structural collapse.

The structure is represented by a discrete model composed of line elements (models of partial joint releases). The line elements admit geometric and

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physical nonlinearities; they can predict the behavior or reinforced concrete members subject to axial and flexural distortions up to failure. The state of the element is characterized by its internal energy.

The state of the system is defined by the work function, a scaler function that contains implicitly all the forces acting on the system. The work function is uniquely defined in terms of the generalized coordinates, which must be related to the equilibrium path (motion) when the system behaves nonlinearly.

The solution process (the determination of the generalized coordinates relative to a prescribed load or a specific time) is based on the minimization of the work function. Stable equilibrium states of the system correspond to points at which the work function assumes a relative minimum.

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This technical report has been reviewed and is approved for publication.

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CONTENTS

SECTION 1

INTRODUCTION

This investigation is concerned with the prediction of the nonlinear response of reinforced concrete structures, including member failures and structural collapse, to static and dynamic loads. The computer program SINGER, the product of this investigation, provides the tool for this prediction. This report describes the mathematical models and the solution process which form the basis of SINGER.

1.1 BACKGROUND

Since it was desired to represent the structure by a discrete model composed of "gross elements," it was natural to select the finite element method to model the structure. However, the selection of the solution process represented a pivotal decision. Two methods were given serious consideration: the step-by-step (STEP) approach, an equilibrium approach in which the structure is represented by a stiffness matrix; and the minimization (MIN) approach, an energy approach in which the structure is characterized by a work function. In both approaches, the solution process initiates at a point where the state of the system is known and proceeds along discrete points of the equilibrium path (motion) of the system.

The STEP approach has been used extensively in the analysis of nonlinear structures and is well documented [e.g., 14,17]*. The central idea of the STEP approach is contained in Newton's method of successive approximations to a real root [15]. It is illustrated in Figure 1a, which depicts the nonlinear equilibrium path of a one-degree-of-freedom

*Numbers in brackets designate references

system. The path is defined by the equilibrium equation

$$p = f(x) \tag{1.1}$$

where p is the applied load, f is the restoring force (a nonlinear function in x), and x denotes the displacement from the unloaded state. The condition of equilibrium corresponding to a specific load $\bar{p}$ is

$$\Delta p = \bar{p} - f(x) = 0 \tag{1.2}$$

where Δp denotes the unbalanced load. The tangential stiffness at any point of the equilibrium path is defined by

$$k_i = \frac{df(x_i)}{dx} \tag{1.3}$$

If Newton's process is applied to the n^{th} trial solution, x_n, and

$$\Delta p_n = \bar{p} - f(x_n) \neq 0 \tag{1.4}$$

the correction to x_n is

$$\Delta x_n = k_n^{-1} \Delta p_n \tag{1.5}$$

Thus, the $n + 1^{st}$ trial solution is

$$x_{n+1} = x_n + \Delta x_n \tag{1.6}$$

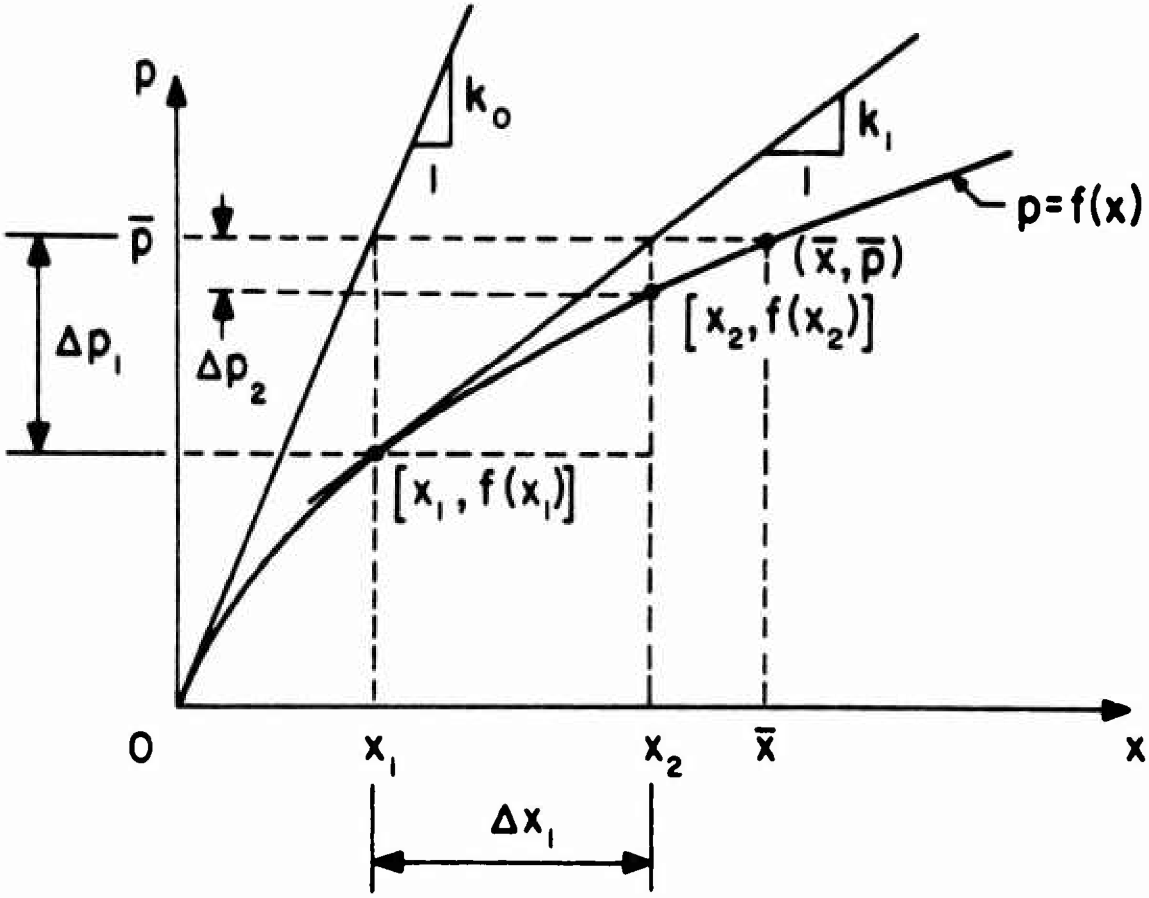
This process is continued until the unbalanced force Δp is sufficiently small.

Two modifications of this process are obtained by using the constant stiffness coefficient

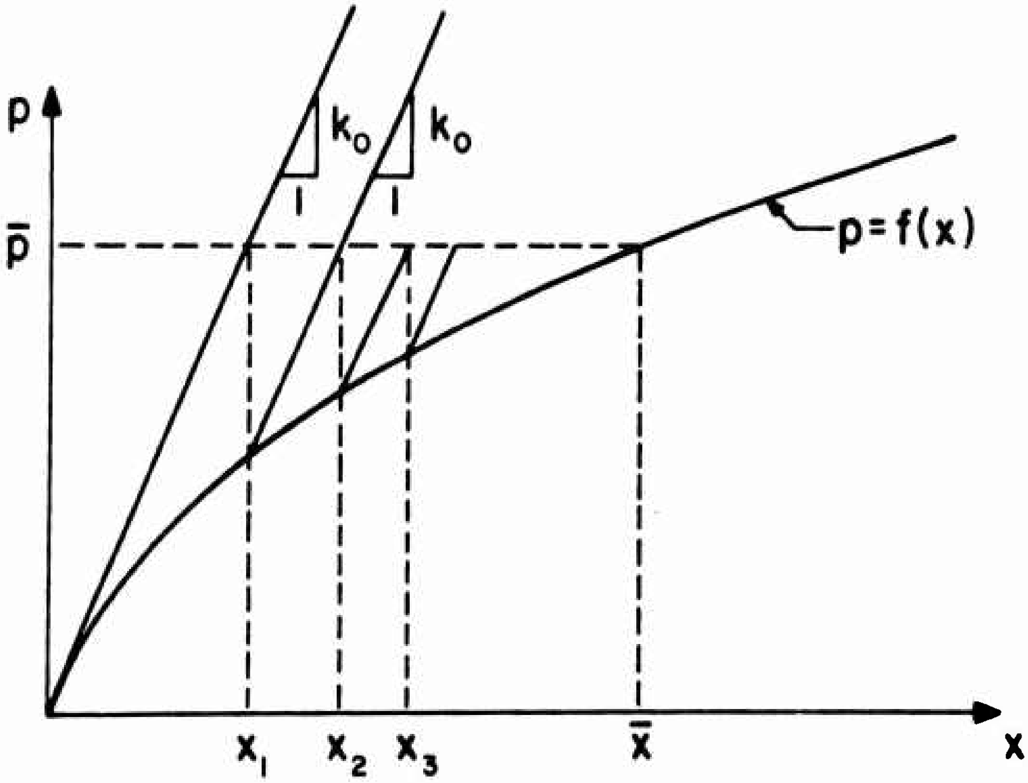
$$k_o = \frac{df(o)}{dx} \tag{1.7}$$

during the entire iterative process (see Figure 1b) or by combining the constant and variable stiffness coefficients in the solution process [17].

The extension of Newton's method to a system with multi-degrees-of-freedom is known as the Newton-Raphson method. On the basis of the finite



(a) VARIABLE STIFFNESS



(b) CONSTANT STIFFNESS

FIG. 1. STEP PROCESS

element method, the governing equations of equilibrium can be expressed in the form [17]

$$p = \int_V \bar{B}^T \sigma \, dV \tag{1.8}$$

where

$$\varepsilon = Bx \tag{1.9}$$

and

$$\delta\varepsilon = \bar{B}\delta x \tag{1.10}$$

In Equations 1.8, 1.9, and 1.10, p and x represent the external generalized force and displacement vectors, respectively; σ and ε denote the stress and strain vectors, respectively (the constitutive laws may be nonlinear); B is the compatibility matrix which may depend on x, in which case $\bar{B} \neq B$; δ signifies a virtual variation; and V denotes the volume of the system. Again the condition of equilibrium for a specific force vector $\bar{p}$ is

$$\Delta p = \bar{p} - \int_V \bar{B}^T \sigma \, dV = 0 \tag{1.11}$$

where Δp is the unbalanced force vector. Analogous to the Newton process, the correction to the n^{th} trial solution, x_n, is

$$\Delta x_n = K_T^{-1} \Delta p_n \tag{1.12}$$

and the $n + 1^{st}$ trial solution is defined by

$$x_{n+1} = x_n + \Delta x_n \tag{1.13}$$

The tangent stiffness matrix K_T in Equation 1.12 is obtained by forming a virtual variation of Equation 1.8 with respect to x; the result can be expressed in the form

$$\delta p = K_T \delta x \tag{1.14}$$

The unbalanced force vector corresponding to any trial solution is evaluated on the basis of Equation 1.11. The solution process is continued until the unbalanced forces are sufficiently small. The modifications of the Newton process are also employed in the Newton-Raphson process.

The MIN approach is based on the property that the work function [7] of the system assumes a relative minimum at a stable equilibrium state. Accordingly, a desired equilibrium state is found by minimization of the work function. Function minimization is accomplished via nonlinear programming techniques. The MIN approach has been employed successfully in the analysis of nonlinear structures [e.g., 2, 5, 9].

The MIN process, which is discussed in more detail in section 3, is illustrated for a two-degree-of-freedom system in Figure 2. The work function W is represented by level curves. Function minimization is based on a modification of Davidon's method [13]. The search for the desired equilibrium state $\bar{x}$ corresponding to the applied load vector $\bar{p}$ initiates at x_o in the direction d_1. The first trial solution x_1 is obtained by minimizing the function W along the direction d_1. A new search direction d_2 is established, and the relative minimum of W with respect to d_2 is found to be x_2 (the search directions are defined by transformations of the gradients of the work function [13]). The iterative process is continued until the components of the gradient of the work function, which correspond to the unbalanced forces, are sufficiently small.

It was decided that both the STEP and MIN approach provide a

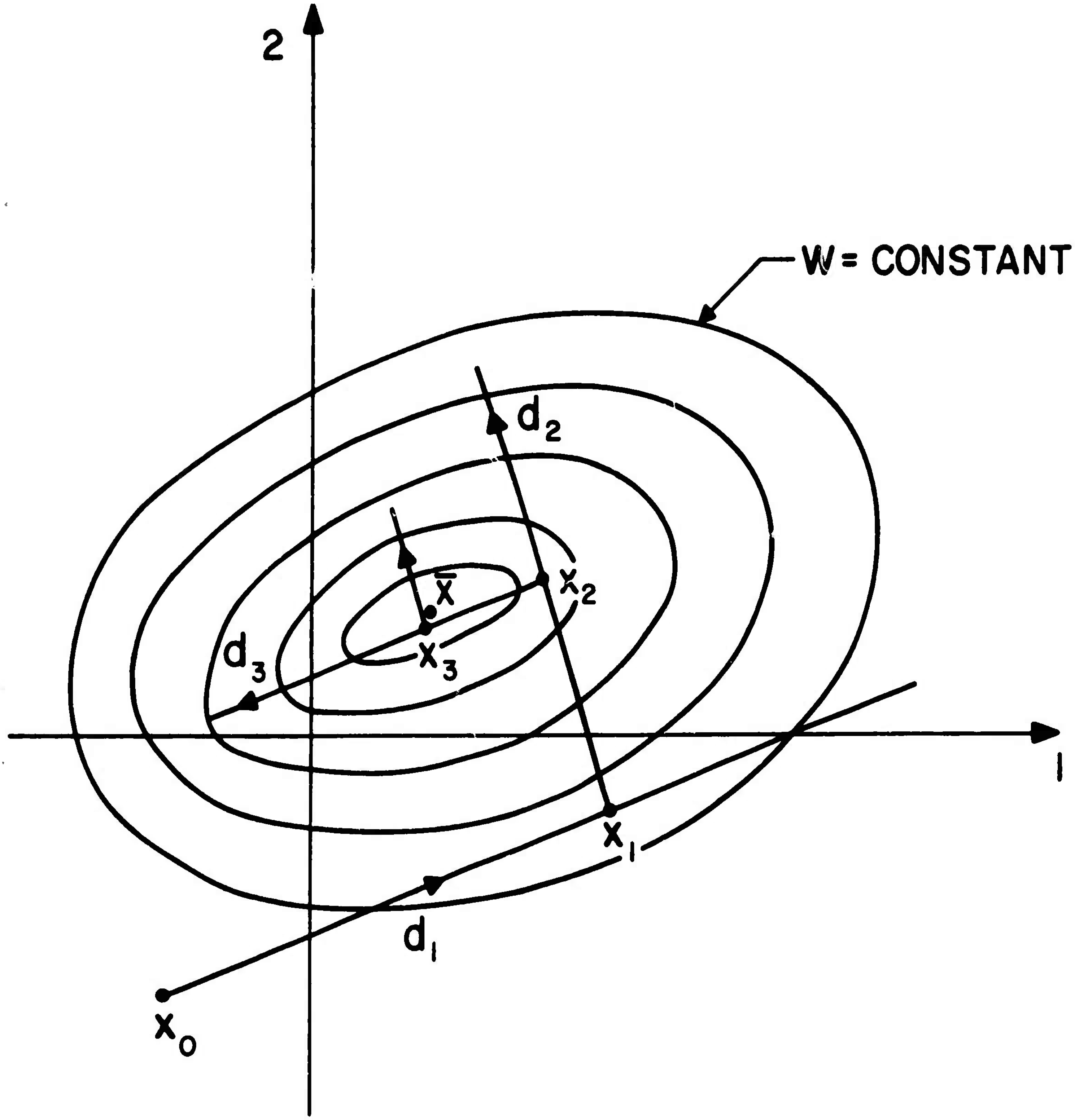


FIG. 2. MIN PROCESS

satisfactory basis for the proposed modeling and solution process. The final selection of the MIN approach was strongly influenced by the following factors:

In the MIN process, the search for an equilibrium state is always based on the actual state of the system corresponding to an assumed displacement configuration. The STEP approach is a quasi-linear approach in which every trial solution is based on the stiffness properties of the system at the beginning of the iteration (Figure 1). Hence, in the MIN approach, decisions are always based on the actual state of the system.

Depending on the choice of the minimization algorithm, substantial storage space savings can be achieved with the MIN process since the structure is represented by a scalar function. However, the Davidon algorithm [13] does require storage space comparable to the STEP process.

The computation and "inversion" of the tangent stiffness matrix in the Newton-Raphson process (Equation 1.12) requires a significant amount of computational effort at each cycle. The alternate approach of a constant stiffness matrix converges only for certain types of nonlinearities [14]. Hence, at least a combination of the constant and variable stiffness matrices is required.

1.2 PURPOSE AND SCOPE

The function of the computer program SINGER is to predict the behavior of plane skeletal reinforced concrete structures in their environ-

ments. Of particular interest is the nonlinear transient response including the possibility of element failures and structural collapse.

SINGER is intended to serve as a tool for the improvement and development of techniques for the assessment of existing protective structures, the design of new systems, and the development of motion environment criteria for internal systems of protective structures.

1.3 METHODOLOGY

The prediction of the performance of the structure in its environment is based on the response of a mathematical model of the structure to actions, which simulate the environment. The analysis process comprises three principal tasks:

1. The formulations of actions, the mathematical models of the environment.
2. The development of a mathematical model of the structure.
3. The formulation of the solution process.

The actions consist of the self-weight of the structure, distributed and concentrated static and dynamic loads, inertia forces, and support motions.

The structure is represented by an assemblage of discrete line elements and springs interconnected at a finite number of points. The line elements are models of straight, prismatic, reinforced concrete members whose longitudinal plane of symmetry corresponds to the plane of loading. The line element is discretized via the finite element method; the internal energy, which characterizes the state of the element, is a function of the element distortion components (three relative end-displacements

and one relative internal-displacement). Springs represent models of joints with partial releases. A concentrated mass is assigned to each degree of freedom of the assemblage. Energy dissipation resulting from inelastic behavior accounts for structural damping.

The line elements admit geometric and physical nonlinearities. Geometric nonlinearities are induced by the coupling of flexural and axial distortions and the formulation of equilibrium for the deformed state of the assemblage. Physical nonlinearities are caused by non-linear constitutive (stress-strain) laws. The springs are assumed to behave linearly.

The behavior of the element is modeled up to the limit of continuous change of state, defined as fracture (e.g., crushing of the compression block constitutes element failure; however, minor discontinuties such as spalling of the concrete cover are modeled).

In the linear domain, the state of the system is completely defined by the generalized coordinates which consist of nodal displacements, relative internal element-displacements, and relative release-displacements. In the nonlinear range, the generalized coordinates must be related to the motion (equilibrium path) of the system to define the state of the system. The origin of the generalized coordinates corresponds to the unstrained state of the system, termed the initial state.

The response of the system to dynamic actions is determined at a discrete number of points in time. The solution process is a closed iterative process within two successive points in time, the time step. The time function of each generalized coordinate is approximated

over the time step by a finite power series whose coefficients are expressed in terms of three known initial conditions, the displacement, velocity, and acceleration at the beginning of the time step, and one unknown end condition, the displacement at the end of the time step. This representation of the time function permits one to express the inertia forces at the end of the time step in terms of the unknown displacements. Consequently, the state of the system at the end of the time step can be completely defined in terms of the corresponding generalized coordinates. For this purpose a work function is introduced, a scalar function of the generalized coordinates, which contains implicitly all the forces acting on the system (applied, inertia, internal). The desired system configuration at the end of the time step is obtained by minimization of the work function, which assumes a relative minimum at the dynamic equilibrium state. The minimization process is a search process in which a system configuration is assumed, the inertia forces are computed and added to the applied external forces, the work function is formulated and tested for a relative minimum. With the aid of the information gained in this test, a new configuration is found, and the process is repeated until the equilibrium imbalance at the end of the time step is sufficiently small.

This solution process can also be employed to obtain the nonlinear response to static loads. Aside from the inertia forces, the difference between the static and dynamic analysis is conceptual. Instead of a time step, a load increment is specified and the corresponding configuration is again obtained by work function minimization.

SECTION 2

MATHEMATICAL MODELS

This section presents mathematical models of plane, skeletal, reinforced concrete structures and their environments.

The model of the structure, the system model, is a discrete model composed of line elements (models of reinforced concrete beam-columns) and springs (models of partial joint releases). The line elements admit geometric and physical nonlinearities; they can predict the behavior of reinforced concrete members subject to flexural and axial distortions up to failure, which is defined as the limit of continuous change of state. The state of the element is characterized by its internal energy. The springs are restricted to linear behavior.

The state of the system is defined by the work function, a scalar function that contains implicitly all the forces acting on the system. The work function is uniquely defined in terms of the generalized coordinates, which must be related to the equilibrium path (motion) when the system behaves nonlinearly (cf. section 2.2.4).

Failure criteria are formulated; they define the domain in which the models are valid and provide the basis for predicting element failure and structural collapse.

2.1 ACTIONS

Actions, mathematical models of the environment, consist of the selfweight of the structure, distributed and concentrated loads, inertia forces, and support motions.

All distributed loads and self-weights are replaced by "equivalent" nodal forces [17]. In the linear range of the element, the equivalent nodal forces caused by transverse member loads are equal in magnitude and opposite in sense to fixed-end forces; this is a consequence of the assumed shape functions (cf. section 2.2.1), which correspond to the homogeneous solution of the differential equation of a beam in flexure. This property does not exist in the nonlinear range where the discrete element forms an approximate representation of the continuum.

Inertia forces are computed on the basis of lumped masses assigned to the nodal degrees-of-freedom. The computation of the lumped masses follows the approach described in reference 12 .

2.2 ELEMENT MODEL

The reinforced concrete beam-column is represented by a gross element model. This means that the element forms a one-dimensional continuum, which is discretized in the modeling process.

The initial state of the element is assumed to be unstrained. Deformations are governed by the fundamental assumption that plane sections remain plane and normal to the deformed reference axis. Consequently, the state at any point of the element is defined by the state of the reference axis. Deformations are limited by the assumption that strains and rotations are small relative to unity. Axial and flexural deformations are modeled explicitly; only a measure of shear distortions and their significance is provided. Inelastic deformations are modeled up to element failure. Structural damping is incorporated through energy dissipation associated with inelastic behavior.

The beam-column effect, the coupling of axial and flexural distortions,

is represented by the corresponding nonlinear term in the strain-displacement relation. The member-force interactions, which are characterized in the concrete literature by behavior models, are also formulated at the micro level. This is natural since the behavior model, a macro model governing the axial load-moment-curvature relations at a section, is completely defined by the following section properties: the strain state, the constituents of the section, and the corresponding constitutive laws. The variability of the neutral axis, a characteristic of reinforced concrete beams subjected to axial and flexural distortions, is modeled by admitting axial strain variations along the reference axis. This feature is illustrated in section 2.2.1.

The state of the element is characterized by its internal energy. Conditions of equilibrium are formulated for the assemblage of elements, the structural system. The modeling process, passing from the continuum to the internal energy expressed in terms of a finite number of distortion components, is depicted schematically in Figure 3 : u and v define the deformed reference axis; $\bar{u}$ is a 4-dimensional element distortion vector whose components represent the relative element displacements; x and y are the coordinates of a point in the element (Figure 5); ε and σ denote strain and stress at a point, respectively; and U signifies the internal energy of the element.

2.2.1 DISCRETIZATION

The reference axis of the element is depicted in Figure 4 . The reference axis must lie in the longitudinal plane of symmetry, the plane of bending, of the element, and all reference axes incident at a joint

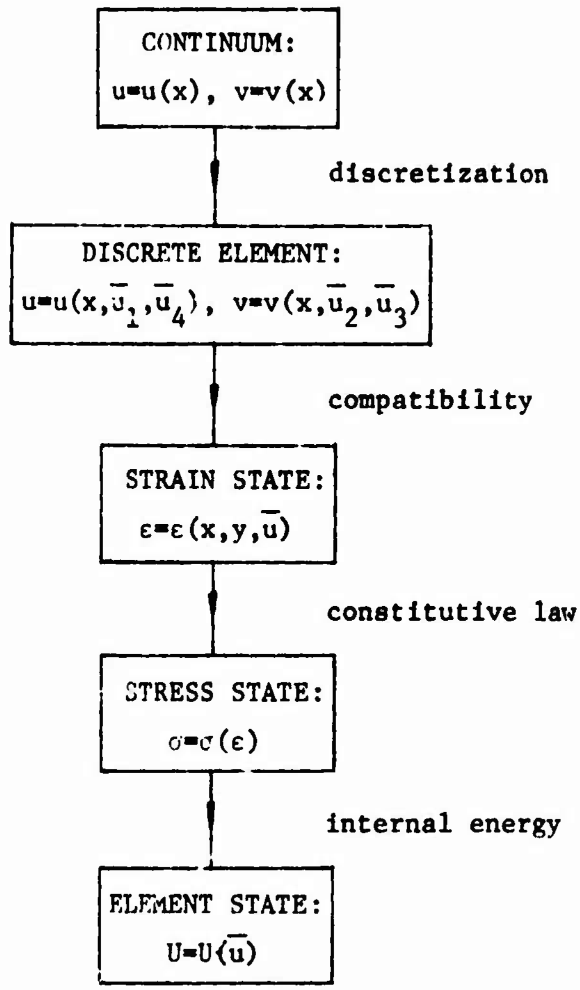


Fig. 3 MODELING PROCESS

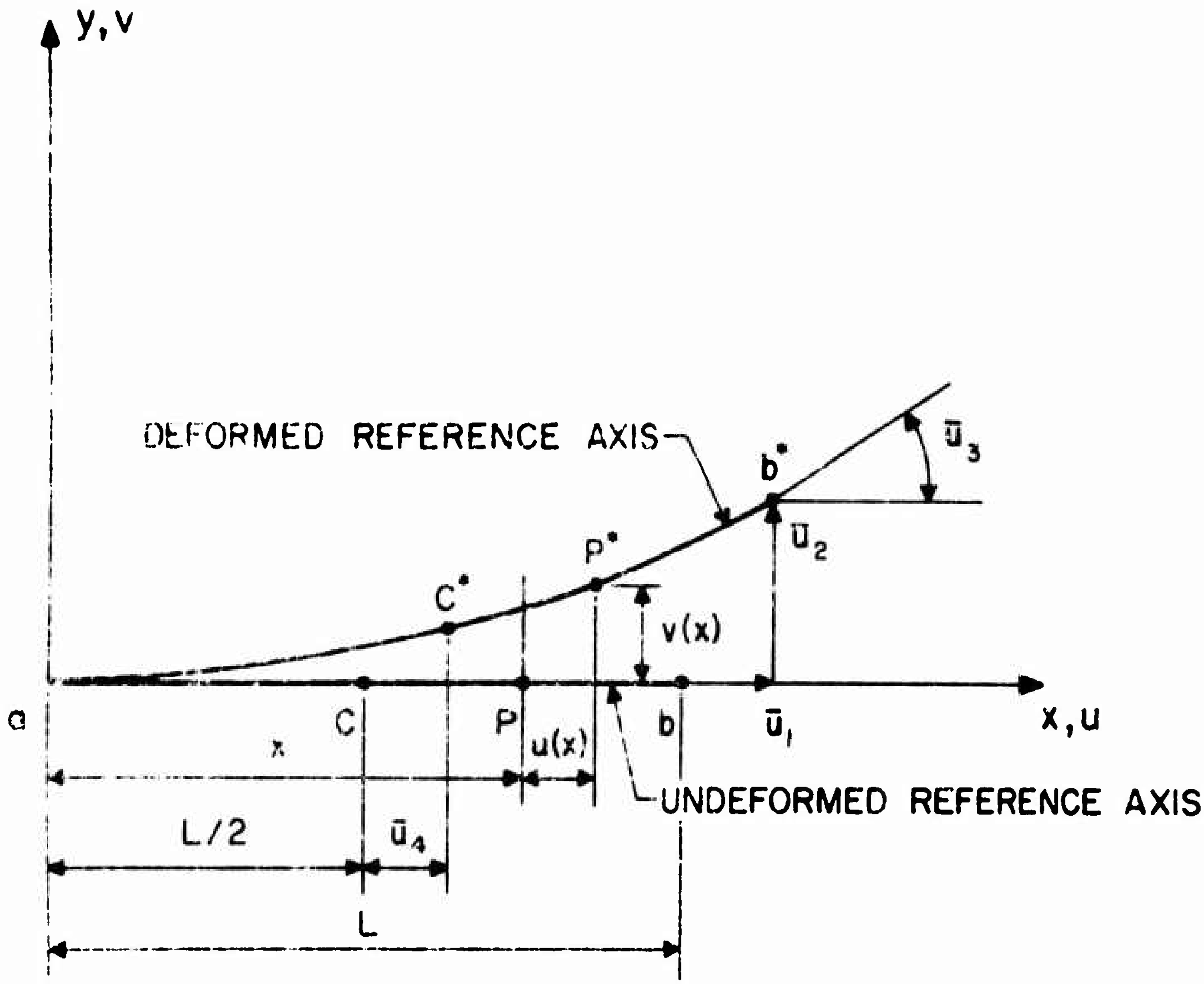


FIG. 4. ELEMENT REFERENCE AXIS

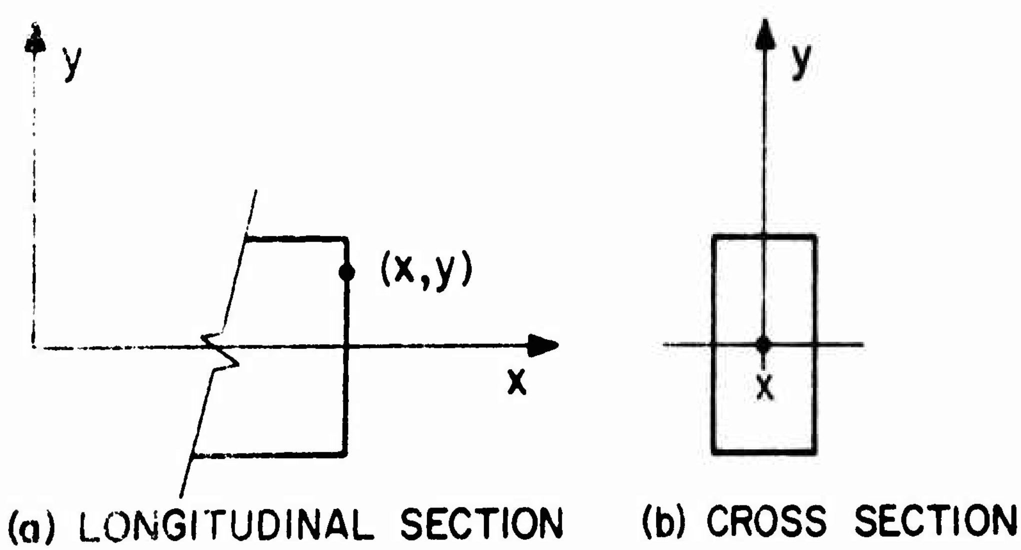


FIG. 5. ELEMENT SECTIONS

must be concurrent. This eliminates the modeling of joint eccentricities. Moreover, the reference axis need only be parallel to a longitudinal edge of the beam; its location in the longitudinal plane of symmetry is arbitrary (see illustrative example on page 20).

The deformation coordinate axes, the x, y-axes in Figure 4 , are defined in section 2.3.1. The transformation of the global joint displacements into the element distortion components $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$ is presented in section 2.3.1. The internal distortion component, $\bar{u}_4$, is prescribed directly in the solution process.

The configuration of the deformed reference axis is expressed in the form

$$u(\xi) = \phi_1(\xi)\bar{u}_1 + \phi_4(\xi)\bar{u}_4 \tag{2.1}$$

$$v(\xi) = \phi_2(\xi)\bar{u}_2 + \phi_3(\xi)\bar{u}_3 \tag{2.2}$$

where

$$\phi_1 = 2\xi^2 - \xi \tag{2.3}$$

$$\phi_2 = -2\xi^3 + 3\xi^2 \tag{2.4}$$

$$\phi_3 = L(\xi^3 - \xi^2) \tag{2.5}$$

$$\phi_4 = 4(-\xi^2 + \xi) \tag{2.6}$$

and

$$\xi = x/L \tag{2.7}$$

For a linear element satisfying the conditions of the elementary flexure theory, Equation 2.2 represents an exact description of the transverse flexural deflection v in terms of the relative end-displacements $\bar{u}_2$, $\bar{u}_3$. For a nonlinear element, the shape functions ϕ_2, ϕ_3

provide only an approximate representation of the flexural response. The introduction of the internal distortion component $\bar{u}_4$ in the longitudinal displacement function, Equation 2.1, permits linear variation in the normal strain along the reference axis (see Equation 2.11). This feature makes it possible to describe the strain state corresponding to a linearly varying neutral axis with respect to any reference axis in the longitudinal plane of symmetry. This property is illustrated in the following example.

Consider the strain state

$$\varepsilon(x,y) = -\varepsilon_o - \frac{2\varepsilon_o}{h} y \left(1 + \frac{x}{L}\right) \tag{2.8}$$

of the beam shown in Figure 6 . The first term on the right-hand side of Equation 2.8 represents a constant normal strain induced by axial compression, and the second term describes a flexural strain that varies linearly with respect to the orthogonal reference axes, x and y; the x-axis coincides with the centroidal axis of the beam; h and L denote the height and length of the beam, respectively. The neutral axis is formed by the straight line passing through points P and Q (Figure 6). Introduce a reference axis that does not coincide with the centroidal axis; e.g., let the location of the reference axis be described by the coordinate transformations

$$y = \bar{y} - \frac{h}{4}, \quad x = \bar{x} \tag{2.9}$$

which places the reference axis a distance h/4 below the centroidal axis (Figure 6). Substitution of Equation 2.9 into Equation 2.8 yields

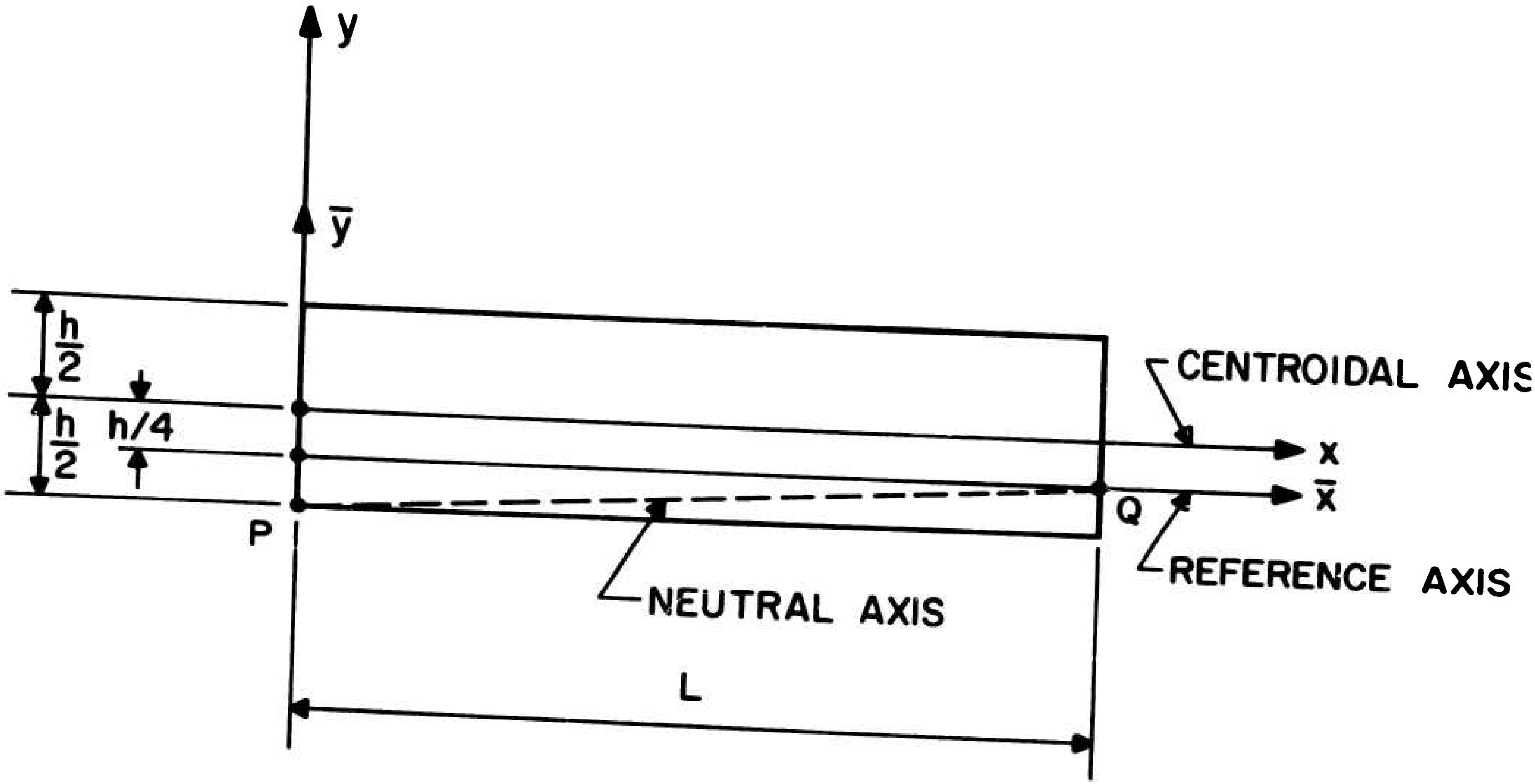


FIG. 6. TRANSLATION OF REFERENCE AXIS

$$\epsilon(\bar{x}, \bar{y}) = - \epsilon_o \left(\frac{1}{2} - \frac{\bar{x}}{L}\right) - 2 \frac{\phi_o}{h} \bar{y}\left(1 + \frac{\bar{x}}{L}\right) \qquad (2.10)$$

A comparison of Equations 2.8 and 2.10 indicates that the translation of the reference axis causes the normal strain to vary linearly along the reference axis but does not alter the form of the flexural strain term.

Hence, a strain state corresponding to a linearly varying neutral axis can be described relative to a reference axis that admits linearly varying normal strains.

2.2.2 COMPATIBILITY

The point-wise deformations of the element are defined by the strain-displacement relation (Figure 5)

$$\epsilon(x,y) = \frac{du}{dx} + \frac{1}{2} \left(\frac{dv}{dx}\right)^2 - y \frac{d^2v}{dx^2} \qquad (2.11)$$

where $\epsilon(x,y)$ is the normal strain (in the x-direction) at any point (x,y); the x-coordinate locates planes normal to the undeformed reference axis, and the y-coordinate locates points in that plane; $u(x)$ and $v(x)$ define the deflections of any point $(x,0)$ on the reference axis in the x and y directions, respectively.

The terms on the right-hand side of Equation 2.11 admit the following geometric interpretations: The first term defines the normal strain induced by axial deformations of the reference axis; the second term represents the contribution of bending of the reference axis to the normal strain [8]; i.e., it accounts for the coupling of axial and flexural distortions; and the third term represents the elementary bending strain.

Equation 2.11 is valid if the strains and rotations are small compared to unity [e.g., 3, 8, 11]. These limitations are characteristic of classical stability investigations leading to conditions of infinitesimal stability (e.g., the Euler buckling load). Equation 2.11 can form the basis of post-buckling investigations provided the strains remain small and the rotations are held small by the division of the element into sub-elements. The same procedure can be employed to model regions of large distortions induced by inelastic deformations.

With the aid of Equations 2.1 and 2.2, the normal strain can be expressed in terms of the element distortion components:

$$\varepsilon(\xi,\eta) = \phi_1' \frac{\bar{u}_1}{L} + \phi_4' \frac{\bar{u}_4}{L} + \frac{1}{2} \left(\phi_2' \frac{\bar{u}_2}{L} + \frac{\phi_3'}{L} \bar{u}_3\right)^2$$

$$- \eta \left(\phi_2'' \frac{\bar{u}_2}{L} + \frac{\phi_3''}{L} \bar{u}_3\right) \qquad (2.12)$$

where

$$\phi_1' = 4\xi - 1 \qquad (2.13)$$

$$\phi_4' = 4(-2\xi + 1) \qquad (2.14)$$

$$\phi_2' = 6(-\xi^2 + \xi) \qquad (2.15)$$

$$\phi_3' = L(3\xi^2 - 2\xi) \qquad (2.16)$$

$$\phi_2'' = 6(-2\xi + 1) \qquad (2.17)$$

$$\phi_3'' = L(6\xi - 2) \qquad (2.18)$$

and

$$\eta = y/L \qquad (2.19)$$

2.2.3 CONSTITUTIVE LAWS

The stress-strain laws governing material behavior are presented in appendix A. They are expressed in terms of piece-wise linear functions such that to every point in the domain (ε) corresponds a unique point in the range (σ), which is determined by the strain history.

The constitutive laws presented model the behavior of concrete (unconfined and confined) and reinforcing steel for monotonic and cyclic loading. The inherent assumptions and limitations are stated.

2.2.4 INTERNAL ENERGY

Energy evaluation represents the pivotal task in the search of the equilibrium state corresponding to a prescribed time (or load). All measures of response (e.g.,displacements, deformations, strains, stresses, energies) are expressed relative to the "initial state," which is the unstrained and unloaded configuration of the system.

Energy evaluation in the context of the solution process means the computation of the total energy of the system for a given displacement state. The internal energy evaluation proceeds as follows: On the basis of Equation 2.11 and appropriate constitutive laws (appendix A), the "internal-energy density," the internal energy per unit volume, is determined. Integration of the internal-energy density over the volume of the element yields the internal energy of the element. The internal energy of the system is equal to the sum of the internal energies of all elements comprising the system (if the system contains release springs, their strain energies must be added).

The principal assumption in the energy computation is that no "load

reversals" occur during a time step; i.e., during the entire time step, the strain at any point in the system is either monotone increasing or monotone decreasing.

Energy evaluations must be conducted numerically. In the elastic range, numerical integration is dictated by the possible variation of cross-sectional properties (e.g. area of compression block) over some region of the element. For instance, an axial load and a varying bending moment cause a varying neutral axis (cf. illustrative example on page 20). In the inelastic range, it is not possible to formulate explicitly the variation of the internal-energy density over the volume of the element.

The numerical energy evaluation is based on the discretization of the energy stored in the element. It involves two principal tasks:

1. The computation of the internal-energy density at a discrete number of points in the element.
2. The integration of the internal-energy density over the volume of the element.

The computation of the internal-energy density during the solution process of a typical time step, from t_1 to t_2, is described with the aid of Figure 7 . t_1 corresponds to the time at which the last equilibrium state of the system has been obtained, and t_2 denotes the time at which the next equilibrium state is sought. The stress-strain curves in Figure 7 govern the behavior of a discrete point of the element. ε_1 and ε_2 denote strains at t_1 and t_2, respectively; both

loading ($\epsilon_2 > \epsilon_1$) and unloading ($\epsilon_2 < \epsilon_1$) cases are illustrated. The internal-energy density at time t_2 is

$$U^*_2 = U^*_1 + U^*_{12} \qquad (2.20)$$

where

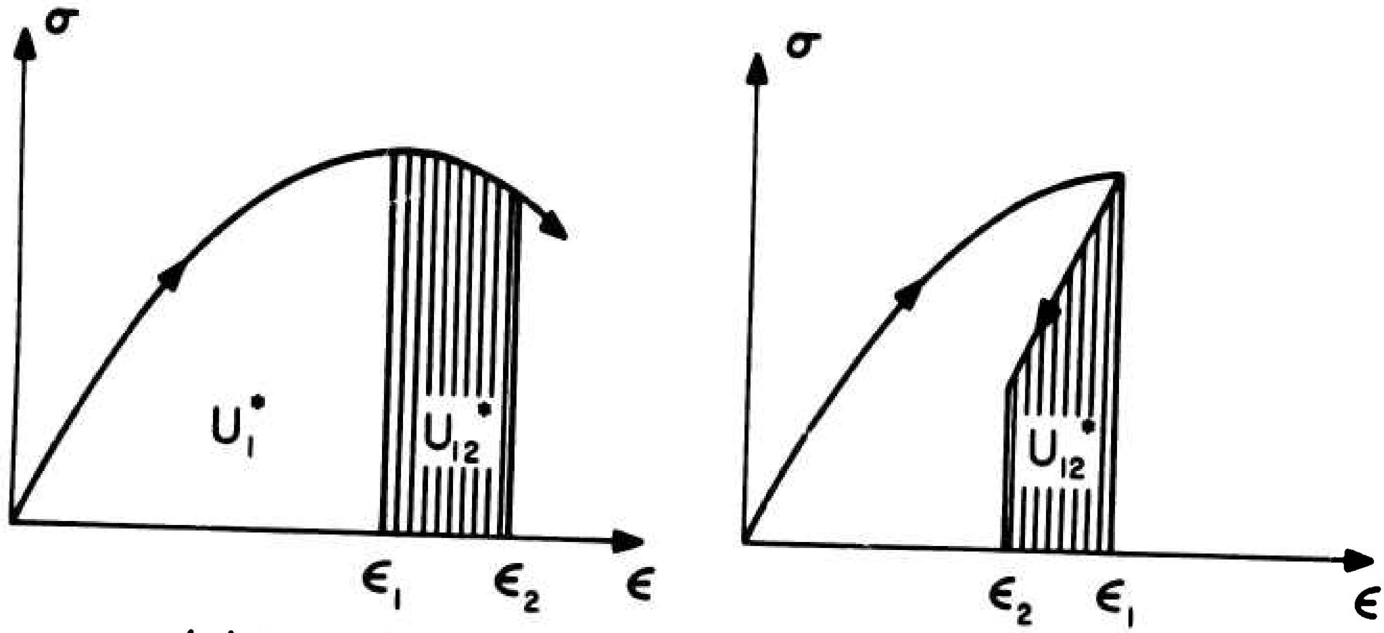
$$U^*_1 = \int_0^{\epsilon_1} \sigma d\epsilon \qquad (2.21)$$

represents the internal-energy density at t_1, and

$$U^*_{12} = \int_{\epsilon_1}^{\epsilon_2} \sigma d\epsilon \quad = \quad \begin{cases} >0 & \text{if} \quad \epsilon_2 > \epsilon_1 \\ <0 & \text{if} \quad \epsilon_2 < \epsilon_1 \end{cases} \qquad (2.22)$$

represents the change in the internal-energy density during the time step t_1, t_2. It follows from Figure 7 that for a given value of strain ϵ_2, there corresponds a unique value of stress. Consequently, the internal-energy density, and hence the internal energy, is uniquely defined by the strain state, which in turn is a unique function of the displacement state. Hence, in the neighborhood of an equilibrium state, the internal energy of the system is a unique function of the generalized coordinates.

In the inelastic range, the internal-energy density consists of a dissipative component U^*_d, which is locked into the material by residual stresses on the microscopic level, and a recoverable component U^*_r, which is released by the material upon unloading (see Figure 8). The dissipative component accounts for structural damping.



(a) LOADING

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FIG. 7. INTERNAL-ENERGY DENSITY

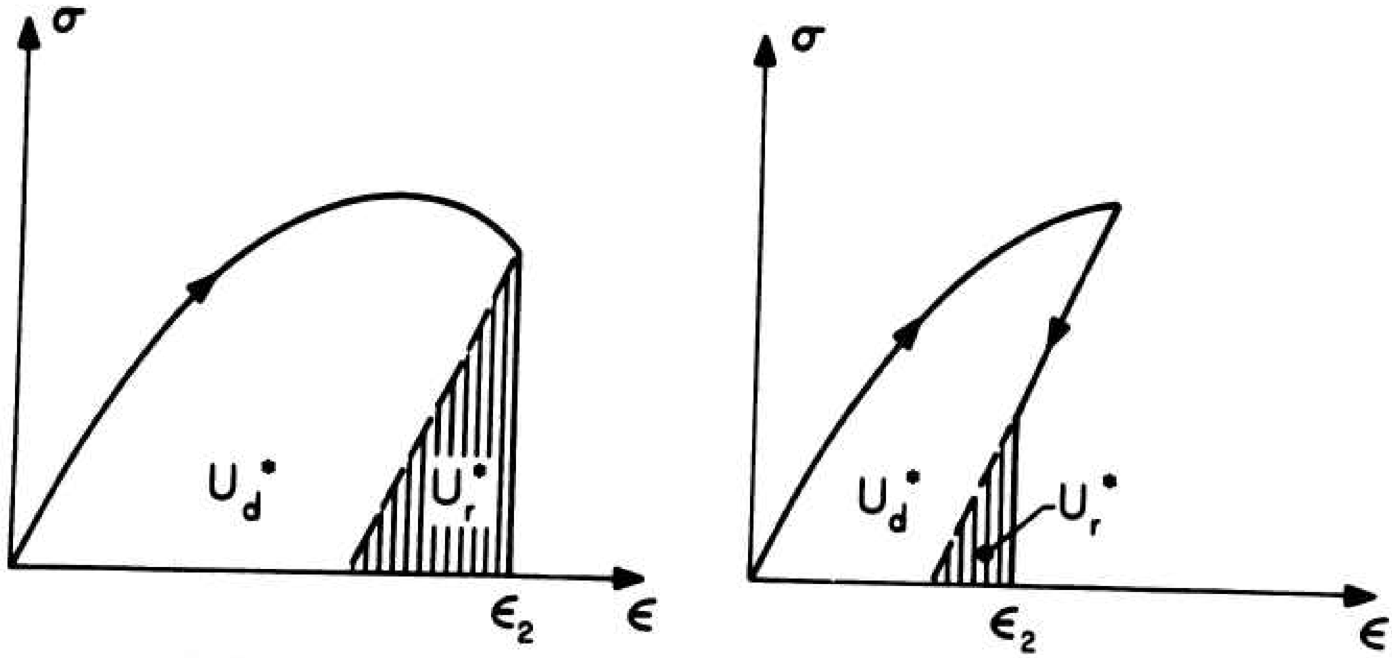

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FIG. 8. DISSIPATIVE & RECOVERABLE ENERGY

The computation of the internal energy of the element is based on the Gaussian quadrature method [17]; the concrete and steel are considered separately. The internal-energy densities are evaluated at discrete points, the Gauss points, and substituted into the Gaussian quadrature formula to yield the energy stored in the element. The Gauss points are distributed in the longitudinal plane of the element as follows: six points (a 2 x 3 rule) are placed in the top and bottom concrete covers, nine points (a 3 x 3 rule) are placed in the concrete between the covers, and three points are placed along the centroidal axis of each steel layer. The accuracy of the energy computation increases with the number of Gauss points per element, which at present is fixed. Hence, it can only be controlled indirectly through the division of the element into sub-elements.

Energy variations govern the behavior of the mathematical model of the structure. The accuracy of response predictions of the structure is limited by the accuracy inherent in the energy evaluations. For this reason internal energies induced by shear distortions are not included in the mathematical model; only estimates of the internal energy caused by shear distortions and measures of the significance of these distortions are provided. The modification of the element model to account for shear distortions introduces uncertainties which may seriously affect the reliability of the model. The sources of uncertainty are identified in the following discussion.

On the basis of the elementary beam theory including shear effects,

the shape functions in Equation 2.2 can be modified to assume the form [12]

$$\phi_2 = \frac{1}{1+\gamma}(-2\xi^3 + 3\xi^2 + \gamma\xi) \tag{2.23}$$

and

$$\phi_3 = \frac{L}{1+\gamma}[\xi^3 - \xi^2 + \frac{\gamma}{2}(\xi^2 - \xi)] \tag{2.24}$$

where

$$\gamma = \frac{12EI}{L^2 A_s G} \tag{2.25}$$

and

$$A_s = \frac{A}{\kappa} \tag{2.26}$$

γ is a measure of the relative importance of shear deformations. In particular, γ is the ratio of the shear deflection to the bending deflection of a fixed-fixed beam subject to a relative end displacement. It is important to recall that Equation 2.25 is based on the assumption that the beam is prismatic, homogeneous, isotropic, and linearly elastic. Accordingly, the symbols in Equations 2.25 and 2.26 are constants for a given beam: E and G denote Young's modulus and the shear modulus of elasticity, respectively; A, A_s, and I define the area, the effective shear area, and the moment of inertia of the cross section, respectively; κ is a shape factor that reflects the variation of the shear stress across the section; and L is the length of the beam.

For an inelastic reinforced concrete beam-column, the quantities in Equation 2.25 are not constants: The moduli E and G vary pointwise over the volume of the uncracked concrete and steel; the section properties A, A_s, and I vary with the longitudinal axis of the beam due to non-

uniform cracking; especially the effective shear area A_s is difficult to define since the shear-stress distribution over a cracked section is not known. In essence, the problem is that Equation 2.25 is defined in terms of macro quantities which at best provide an indirect description of the state of an inelastic reinforced concrete beam-column. The same difficulty is encountered in the formulation of the shear energy which is defined by the relation

$$U_s = \int_0^L \frac{V^2 dx}{2A_s G} \qquad (2.27)$$

or

$$U_s = \frac{V^2 L}{2A_s G} \qquad (2.28)$$

since the shear force V is constant in the element model.

In view of the uncertainties inherent in the prediction of shear effects, they are not modeled explicitly; only a measure of the significance of shear distortions is provided on the basis of Equation 2.25, and an estimate of the internal energy induced by shear distortions is made on the basis of Equation 2.28. In the evaluation of Equations 2.25, 28, E and G are assumed to be elastic and κ is set equal to 1.20.

2.2.5 STRESS RESULTANTS

The element end-forces, which act at the reference axis (cf. Figure 9), are computed on the basis of the following formulas:

$$f_{b1} = \int_{A(L)} \sigma(L,y)dA \qquad (2.29)$$

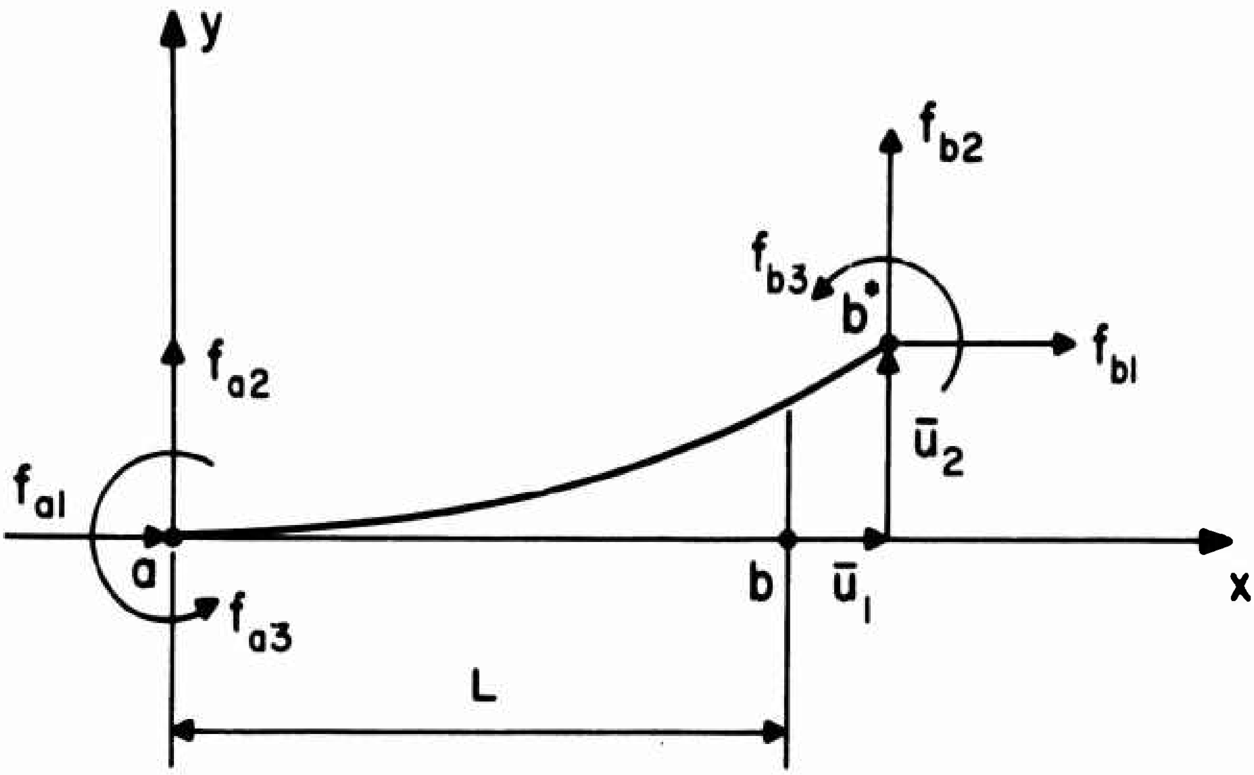

FIG. 9. ELEMENT FORCES
(STRESS RESULTANTS)

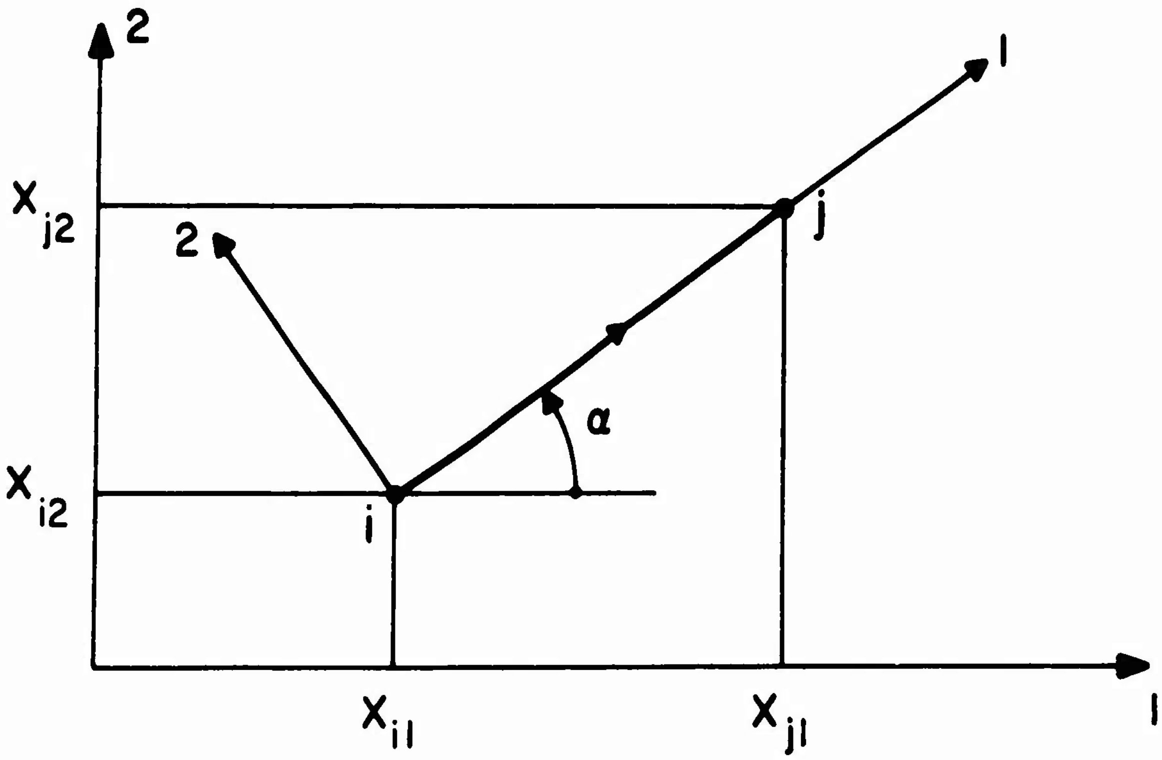

FIG. 10. INITIAL ELEMENT CONFIGURATION

$$f_{b3} = -\int_{A(L)} y\sigma(L,y)\,dA \tag{2.30}$$

$$f_{a3} = \int_{A(0)} y\sigma(0,y)\,dA \tag{2.31}$$

$$f_{b2} = (-f_{b3} - f_{a3} + f_{b1}\bar{u}_2)/L \tag{2.32}$$

$$f_{a1} = -f_{b1} \tag{2.33}$$

$$f_{a2} = -f_{b2} \tag{2.34}$$

where f_{ai}, f_{bi}, $i = 1, 2, 3$, are the element forces at the a & b-end, respectively; and A(0), A(L) represent the cross-sectional areas at the a & b-end, respectively.

2.3 SYSTEM MODEL

The system model is a mathematical representation of plane, skeletal, reinforced concrete structures. It is an assemblage of line elements interconnected at a finite number of nodes. The elements are assumed to be rigidly connected at the nodes unless partial or complete releases are specified.

In the linear domain, the state of the system is completely defined in terms of the generalized coordinates which consist of nodal displacements, internal-element distortion components, and relative displacements at releases. In the nonlinear domain, the generalized coordinates must be related to the equilibrium path (motion) of the system to define the state of the system (see section 2.2.4). In the "initial state," the

generalized coordinates are zero.

There is no restriction on the magnitude of the generalized coordinates per se; however, relative displacements, such as the relative displacements of nodes linked by an element, are limited by the small deformation requirements of the element (cf. section 2.2.2). Violations of these limitations can be resolved through the insertion of additional nodes, i.e., through the subdivision of elements.

The following sections are concerned with compatibility and stability of equilibrium of the system.

2.3.1 COMPATIBILITY

This section relates nodal displacements with relative element displacements, called element distortion components. In the derivation of these components, four orthogonal, right-handed, Cartesian coordinate systems are employed; they are called global, local, joint, and deformation systems. The deflections are positive if they take place in the positive direction of the 1, 2-axes; the positive sense of rotations about the 3-axis is determined by the right-hand rule.

The global and local systems correspond to the coordinate systems used in linear matrix analysis (Figure 10). Joint coordinates and joint properties (e.g., forces and displacements) are expressed in global coordinates and denoted by capital letters. Local axes define the orientation of the undeformed element: the 1-axis coincides with the reference axis, and the 2 & 3-axes correspond to principal axes of the cross-section. The 1-axis specifies the direction of the element; the element in Figure 10 goes from joint i to joint j. Local vectors are

identified by lower-case letters.

The transformation of a two-dimensional global vector Y into a two-dimensional local vector y is defined by the matrix A:

$$y = AY \tag{2.35}$$

where

$$A = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \tag{2.36}$$

and

$$c = \cos\alpha, \quad s = \sin\alpha \tag{2.37}$$

It follows from Fig. 10 that

$$c = \Delta X_1/L, \quad s = \Delta X_2/L \tag{2.38}$$

where

$$\Delta X_1 = X_{j1} - X_{i1}, \; \Delta X_2 = X_{j2} - X_{i2} \tag{2.39}$$

and the initial element length

$$L = (\Delta X_1^{\,2} + \Delta X_2^{\,2})^{\frac{1}{2}} \tag{2.40}$$

The joint and deformation reference frames are moving frames of reference rigidly attached to the joint at the origin of the element (Fig. 11). In the initial state, the joint coordinate system coincides with a global coordinate system originating from that joint, and the deformation coordinate system coincides with the local coordinate system.

Vectors expressed in joint and deformation coordinate systems are identified by barred capital and barred lower-case letters, respectively. Since the joint and deformation reference frames are fixed relative to each other, corresponding vectors are transformed by the A matrix; i.e.,

$$\bar{y} = A\bar{Y} \tag{2.41}$$

where $\bar{y}$ and $\bar{Y}$ are vectors expressed in deformation and joint coordinates, respectively.

The global-joint transformation is given by

$$\bar{Y} = BY \tag{2.42}$$

where

$$B = \begin{bmatrix} c_i & s_i \\ -s_i & c_i \end{bmatrix} \tag{2.43}$$

$$c_i = \cos U_{i3}, \quad s_i = \sin U_{i3} \tag{2.44}$$

and U_{i3} is the rotation of joint i about the 3-global axis.

If follows from Eqs. 2.41 & 42 that the global-deformation transformation is defined by

$$\bar{y} = CY \tag{2.45}$$

where

$$C = AB \tag{2.46}$$

The derivation of the element distortion components follows directly from Fig. 11 . The relative member-end rotation

$$\bar{u}_3 = U_{j3} - U_{i3} \tag{2.47}$$

where U_{j3} and U_{i3} are the rotations of the joints j and i, respectively. The relative member-end deflections $\bar{u}_1$, $\bar{u}_2$ are expressed in matrix form

$$\bar{u} = C\Delta X^* - \bar{d} \tag{2.48}$$

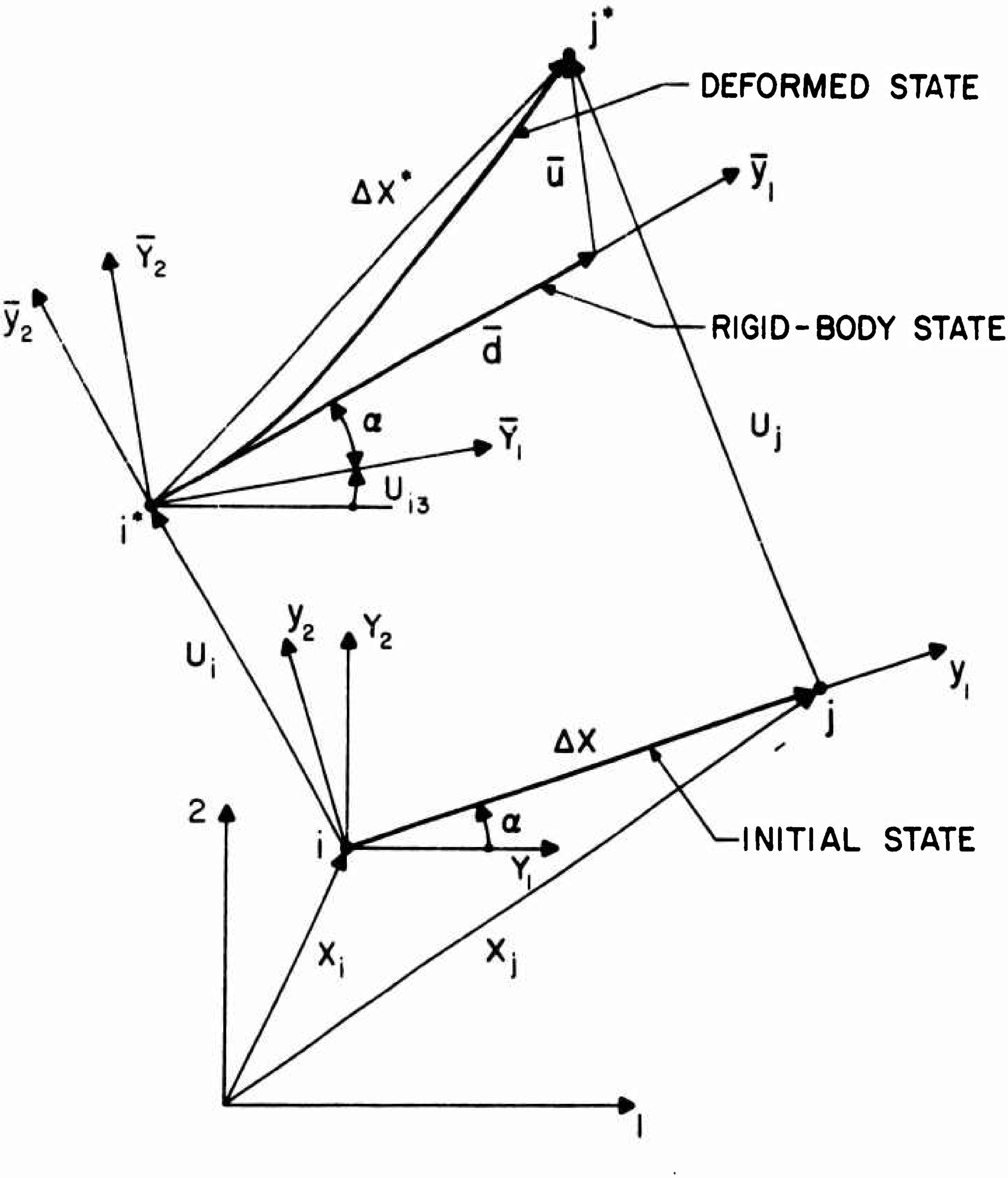

FIG. 11. ELEMENT DISTORTION

where

$$\bar{u} = \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \end{bmatrix} \tag{2.49}$$

$$\Delta X^* = \Delta X + \Delta U \tag{2.50}$$

$$\Delta X = X_j - X_i = \begin{bmatrix} X_{j1} - X_{i1} \\ X_{j2} - X_{i2} \end{bmatrix} = \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \end{bmatrix} \tag{2.51}$$

$$\Delta U = U_j - U_i = \begin{bmatrix} U_{j1} - U_{i1} \\ U_{j2} - U_{i2} \end{bmatrix} = \begin{bmatrix} \Delta U_1 \\ \Delta U_2 \end{bmatrix} \tag{2.52}$$

and

$$\bar{d} = \begin{bmatrix} L \\ 0 \end{bmatrix} = A\Delta X \tag{2.53}$$

In Eqs. 2.51-53, X_i, X_j are the joint position vectors; U_i, U_j are the joint deflection vectors: and $\bar{d}$ defines the rigid-body motion of the element. With the aid of Eqs. 2.46, 50, & 53, Eq. 2.48 can be reduced to a form suitable for numerical evaluation:

$$\bar{u} = A(D\Delta X + B\Delta U) \tag{2.54}$$

where

$$D = B - I = \begin{bmatrix} -2s_{i2}^2 & s_i \\ -s_i & -2s_{i2}^2 \end{bmatrix} \tag{2.55}$$

$$s_{i2} = \sin(U_{i3}/2) \tag{2.56}$$

and I is the identity matrix. For infinitesimal displacements (i.e., $s_i = U_{i3}$, $c_i = 1$, $s_{i2}^2 = 0$, $U_{i3}\Delta U_1 = U_{i3}\Delta U_2 = 0$), Eq. 2.54 reduces to

$$\ddot{u} = A(E\Delta X + \Delta U) \tag{2.57}$$

where

$$E = \begin{bmatrix} 0 & U_{i3} \\ -U_{i3} & 0 \end{bmatrix} \tag{2.58}$$

2.3.2 STABILITY OF EQUILIBRIUM

As described in section 3.2, the search for the equilibrium state corresponding to a set of prescribed forces is governed by the principle of least action; i.e., at an equilibrium state the energy function assumes a relative minimum. Thus, if an equilibrium state is found, it is a stable equilibrium state.

2.4 FAILURE CRITERIA

An assemblage of elements may experience element and system failure. Fracture, the limit of continuous change of state [4], defines element failure. System failure means collapse of the assemblage.

2.4.1 ELEMENT FAILURE

"Structure-sensitive" properties of a material, such as the fracture strength, are essentially determined by local imperfections in the group structure of the material; consequently, they exhibit a considerably greater degree of variability than "structure-insensitive" properties, such as elastic constants [4]. Freudenthal based this

explanation of material behavior on statistical principles.

Although the literature reveals significant variations in the fracture strength of concrete and reinforced concrete elements, the corresponding strength criteria are seldom based on probabilistic models; i.e.,they do not deal with these inherent uncertainties explicitly. In conventional design, the problem of uncertain failure strengths is usually resolved by avoiding such failures rather than by predicting them. The underlying philosophy is to produce ductile structures. For instance, the ultimate moment of an underreinforced concrete beam is governed by the yield strength of the steel. Consequently, the significant variability of the crushing strength of the concrete has little affect on the ultimate flexural strength of the reinforced concrete beam.

In this project, the complete structural response to actions (including system failure) must be predicted. Under static actions, system instability without element failure is possible (e.g., the formation of a collapse mechanism) but perhaps not probable. In the dynamic state, it may not be possible to predict the collapse of the system until the collapse process has been initiated, in which case element failure is probable. In any event, element failure criteria are required.

Since fracture appears to be a probabilistic phenomenon which is not modeled explicitly, it is monitored via lower-bound criteria. When the possibility of fracture is detected, the user must decide whether to base element failure on the conservative lower-bound

criterion or to modify the criterion to yield more probable failure predictions (see appendix B). This procedure requires the user to recognize and deal with the uncertainties inherent in failure criteria.

Element failure criteria are resolved, according to the failure mechanisms, into micro and macro criteria. Micro criteria are formulated on the basis of explicit states at a point, such as the strain state. Macro criteria are expressed in the form of empirical relations, involving stress-resultants and element properties.

Micro criteria predict primary failures, such as crushing and cracking of concrete and fracture of steel, induced by excessive normal strains. The normal strains are caused by flexural and axial distortions. Crushing of concrete may occur in the compression zone of unconfined concrete; it may also take place in conjunction with compression steel "buckling" in confined concrete. Cracking may lead to failure if it initiates in an unreinforced region of a beam in flexure or if the entire cross-section is in tension. Fracture of steel is mainly associated with very light reinforcement.

Macro criteria are concerned with shear-flexure failures [1] which are precipitated by the formation of a diagonal tension crack; the resulting failures are called diagonal-tension, shear-compression, and shear-tension failures. The nominal average shear stress is used as a measure of the diagonal tension strength. For unreinforced webs the occurrence of a diagonal tension crack is regarded as element failure. Although diagonal tension cracks tend to stabilize in short and intermediate-length beams, the crack stabilization mechanism is not well enough understood to warrant utilization of the reserve

strength associated with shear-compression and shear-tension failures. For beams with appropriate web reinforcement, the web reinforcement assures the stabilization of the diagonal tension crack; however, yielding of the web reinforcement can lead again to the type of shear failures experienced by the unreinforced beam.

A classification of all possible failure modes is presented in appendix B. In addition, lower bound criteria are stated, and modifications are formulated for the selection of more probable failure criteria.

2.4.2 SYSTEM FAILURE

System failure can be linked to instability of equilibrium. Stability is the property of equilibrium to sustain disturbances. This means that a stable system remains functional in the perturbed state. Degree of stability of equilibrium is a measure of the disturbances an equilibrium state can sustain [6]. If an equilibrium state is unstable relative to a particular disturbance, the degree of stability is zero.

The solution process employed in this analysis converges only to stable equilibrium states. Hence, the problem is not to ascertain stability of equilibrium but to predict whether an equilibrium state exists for a prescribed set of actions. The concept of degree of stability of equilibrium serves as a basis for this prediction. The "average curvature" of the work function at the equilibrium state is selected as a measure of degree of stability of equilibrium. The computation of the average curvature is based on the values of the curvatures of the work function at the equilibrium state in the

direction of the generalized coordinates. The average curvature is not likely to be zero at an unstable equilibrium state since equilibrium is unstable if the minimum principal curvature is zero. However, the rate of change of a load parameter with respect to the average curvature approaches zero at an unstable equilibrium state. Hence, this rate of change is an indicator of the imminence of instability.

The relation between degree of stability of equilibrium and load level is depicted in Figure 12 ; for the single-degree-of-freedom system, the curvature of the work function at the equilibrium state does approach zero at the limit load, p*. The continuous curve over the domain $0 \leq x < x^*$ represents stable equilibrium states, and the broken curve over the domain $x > x^*$ represents unstable equilibrium states. The decrease in degree of stability of equilibrium with increasing load is illustrated by the work-function curves corresponding to the equilibrium states, x_1, x_2, x^*. The respective curvatures at the equilibrium points decrease monotonically to zero. For a load in excess of the limit load, e.g., $p = p_3$, no equilibrium state exists, and the solution process employed in this study cannot converge.

As the unstable equilibrium state of the system is approached, a load increment could easily push the load beyond the limit load. To prevent a lengthy search for an equilibrium state that does not exist, the solution process is terminated after the deviation from the last equilibrium state exceeds a prescribed bound.

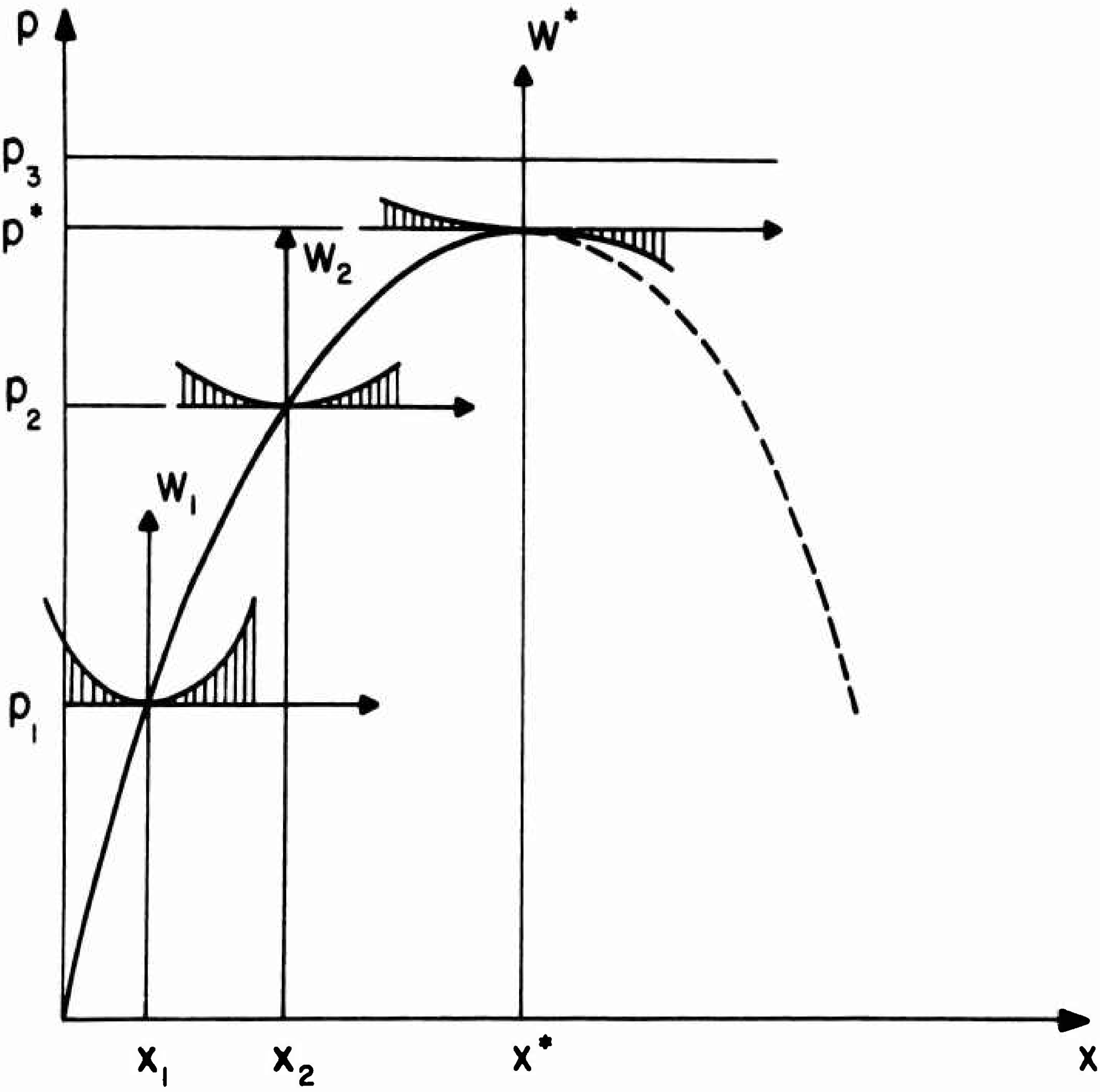

FIG. 12. DEGREE OF STABILITY

2.5 LIMITATIONS

The principal limitations and approximations of the mathematical model of the skeletal reinforced concrete structure are summarized below:

1. The element model is subject to the standard limitations associated with the discretization approach of the finite element method (e.g.,internal element displacements are expressed approximately in terms of the nodal displacements; distributed loads are replaced by "equivalent" nodal forces).
2. Plane sections are assumed to remain plane and normal to the deformed reference axis of the reinforced concrete beam. This assumption appears to be reasonable up to the formation of diagonal tension cracks of unreinforced webs [16], which represent limits of continuous change of state of the element.
3. Normal strains and rotations are assumed to be "small" in the sense that their squares are negligible with respect to unity [3, 11]; i.e., they are regarded to be infinitesimals. These limitations are acceptable since the fracture strains of the materials modeled in this project meet this requirement, and the rotations can be controlled through element subdivision. Shear distortions are not modeled explicitly; the indication is that a modification of the gross element model to include shear deformations is likely to impair the quality of the model.
4. The constitutive laws governing material behavior are described by deterministic models. Consequently, they represent at

best the statistical mean of the material properties and do not reflect the significant randomness characteristic of some properties such as the fracture strength.

5. Energy computation is based on the assumption that no "load reversals" occur during a time (or load) increment of the solution process. Moreover, the computation of the internal energy is based on the evaluation of the internal energy densities at a discrete number of points in the beam element. This introduces another discretization error, which vanishes only in the limit.
6. Element failures precipitated by material fractures are inherently random phenomena which can only be monitored by lower bound criteria in a deterministic analysis. In the event that structural collapse is strongly influenced by element failures (as in contrast to the formation of a "plastic" collapse mechanism), the quality of this prediction by deterministic methods is questionable.

SECTION 3

RESPONSE

The response of the system model to actions is sought at a discrete number of points in time. The solution process, formulated by Melosh and Kelley [9], is a closed iterative process within two successive time points: The state of the system is assumed to be known at the beginning and is sought at the end of the time step. Thus, if the state of the system is known at one point in time, the response determination proceeds like a chain reaction through successive discrete points.

The solution process comprises two fundamental concepts:

1. discretization of motion
2. work-function minimization.

The motion, time functions of the generalized system coordinates, is discretized via the finite element method [17]; this process is analogous to Newmark's β-method [10]: Each displacement function is completely defined over a time step by three initial conditions, which are known, and one end condition, which is the desired displacement at the end of the time step. The work function [7], a scalar function that contains implicitly all the forces acting on the system (applied, inertia, internal), is expressed in terms of the unknown system coordinates at the end of the time step. The desired system configuration is obtained by minimization of the work function, which assumes a relative minimum at the dynamic equilibrium state.

Function minimization is based on Stewart's modification of Davidson's method [13]. A measure of the quality of the response

predictions is provided through error controls linked to automatic time-step selections.

3.1 DISCRETIZATION OF MOTION

The time domain is subdivided into time segments Δt, and the displacement functions are approximated over each subdomain by a finite power series of the form

$$x_i(t) = a_{i0} + a_{i1}t + a_{i2}t^2/2 + a_{i3}t^3/6, \quad 0 \leq t \leq \Delta t \tag{3.1}$$

where x_i represents the i^{th} generalized system coordinate, and t is the normalized time coordinate. The constant coefficients in Eq. 3.1 are determined on the basis of the following end conditions:

$$x_{ai} = x_i(0) \tag{3.2a}$$

$$\dot{x}_{ai} = \frac{d}{dt} x_i(0) \tag{3.2b}$$

$$\ddot{x}_{ai} = \frac{d^2}{dt^2} x_i(0) \tag{3.2c}$$

$$x_{bi} = x_i\,(\Delta t) \tag{3.2d}$$

where x_{ai}, $\dot{x}_{ai}$, $\ddot{x}_{ai}$ denote the displacement, velocity, acceleration, respectively, at the beginning of the time step, and x_{bi} denotes the displacement at the end of the time step. It follows from Eqs. 3.1 and 3.2 that the displacement and acceleration functions can be expressed over the domain [0, Δt] in the form

$$x_i(t) = x_{ai} + \dot{x}_{ai}t + \ddot{x}_{ai}t^2/2 + \beta_i t^3/6 \tag{3.3}$$

$$\ddot{x}_i(t) = \ddot{x}_{ai} + \beta_i t \tag{3.4}$$

where

$$\beta_i = 6(x_{bi} - x_{ai} - \dot{x}_{ai}\Delta t - \ddot{x}_{ai}\Delta t^2/2)/\Delta t^3 \tag{3.5}$$

3.2 WORK-FUNCTION MINIMIZATION

Conditions of dynamic equilibrium are established on the basis of the principle of virtual work, which states that the vanishing of the virtual work for all possible virtual displacements represents a sufficient condition of equilibrium; i.e.,

$$\delta W = 0 \tag{3.6}$$

for all independent virtual displacements is a sufficient condition of equilibrium. The total virtual work can be expressed as

$$\delta W = \delta W_e - \delta U \tag{3.7}$$

where δW_e represents the virtual work of external forces and δU denotes the first variation in the internal energy of the system:

$$\delta W_e = \delta x_b^T p_b \tag{3.8}$$

$$\delta U = \delta x_b^T r_b \tag{3.9}$$

$$p_b = f_b + f_b^e - m\ddot{x}_b \tag{3.10}$$

and the i^{th} component of r_b is

$$r_{bi} = \frac{\partial U}{\partial x_i(\Delta t)} \tag{3.11}$$

The subscript b in Eqs. 3.8-11 signifies that the corresponding variables are evaluated at the end of the time step, at $t = \Delta t$; x_b , $\ddot{x}_b$ denote the generalized displacement, acceleration vectors, respectively; m is a diagonal mass matrix; p_b is the generalized external force vector, which consists of the applied, f_b, equivalent, f_b^e, and inertia, $-m\ddot{x}_b$, force vectors; r_b represents the generalized internal force vector, whose components are the partial derivatives of the internal energy with respect to the generalized coordinates. The superscript T signifies transposition. Eqs. 3.6-9 lead to the condition

$$\delta x_b^T \, (p_b - r_b) = 0 \tag{3.12}$$

which yields the equilibrium equation

$$p_b - r_b = 0 \tag{3.13}$$

In the vicinity of the equilibrium state corresponding to the beginning of the time step, U is a function of the generalized coordinates (cf. section 2.2.4). Moreover p_b is a function of x_b by virtue of Eqs. 3.4, 5. Thus, the equilibrium equation, Eq. 3.13, is a function of x_b.

The unknown generalized coordinates at the end of the time step are not obtained by direct solution of Eq. 3.13 but by minimization of the corresponding work function

$$W(x_b) = x_b^T p_b - U(x_b) \tag{3.14}$$

The stationary condition, Eq. 3.6, which leads to the equation of dynamic equilibrium is also a minimum condition. On the basis of the principle of least action [7], the work function W assumes a relative minimum at x_b, the solution of Eq. 3.13.

3.3 PROCESS ERRORS

There are essentially two sources of error in the solution process [9]: truncation error and iteration error. The truncation error is induced by the approximate representation of the displacement function over a time step by a finite power series. The truncation error decreases with the size of the time step and vanishes in the limit; hence, it can be controlled by varying the length of the time step. The iteration error arises in the minimization process, which converges in the limit to the exact solution. Hence, the iteration error can be made arbitrarily small by a sufficiently large number of iterations.

The force imbalance at the mid-point of the time step is selected as a basis for a measure of the truncation error. It follows from Eq. 3.13 that the unbalanced i^{th} generalized force component

$$\psi_i(t) = p_i(t) - r_i(t) \ , \quad 0 \leq t \leq \Delta t \tag{3.15}$$

The relation

$$e_i(t) = [\psi_i(t) x_i(t)]/W(t) \tag{3.16}$$

transforms the force imbalance into a relative energy imbalance.

Denote

$$e_a = \max|e_i(0)| \tag{3.17a}$$

$$e_{ab} = \max|e_i(\Delta t/2)| \tag{3.17b}$$

$$e_b = \max|e_i(\Delta t)|,\ i = 1, 2, \ldots, n \tag{3.17c}$$

e_a, e_b constitute measures of the iteration error at the beginning and end of the time step, respectively, and e_{ab} is a measure of the truncation and iteration errors at the mid-point; n is the number of generalized coordinates. If one assumes that the iteration error varies linearly over the time step, a measure of the truncation error is obtained in the form (cf. Fig. 13)

$$e_T = e_{ab} - (e_a + e_b)/2 \tag{3.18}$$

The length of the time step is governed by the following inequality

$$e_\ell \leq e_T \leq e_u \tag{3.19}$$

where e_ℓ and e_u define lower and upper bounds on the truncation error measure, respectively. The time step is increased if $e_T < e_\ell$ and decreased if $e_T > e_u$. The lower bound is imposed to assure computer accuracy; i.e., to assure that the time step is large enough to produce measurable changes in the response. The relation between computer error, truncation error, and step length is depicted in Fig. 14 . The accuracy of the solution process is apparently insensitive to variations in Δt over the domain $(\Delta t_1, \Delta t_2)$. The most economical step is near Δt_2.

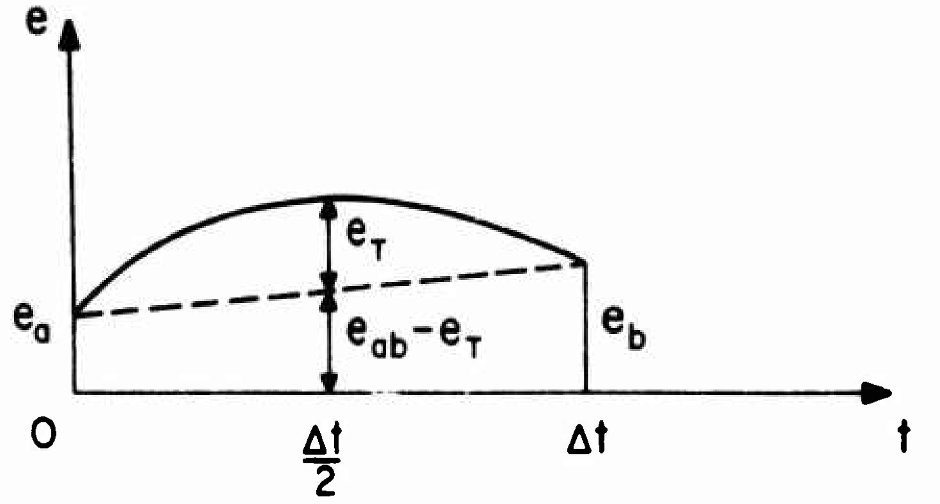

FIG. 13. TRUNCATION & ITERATION ERRORS

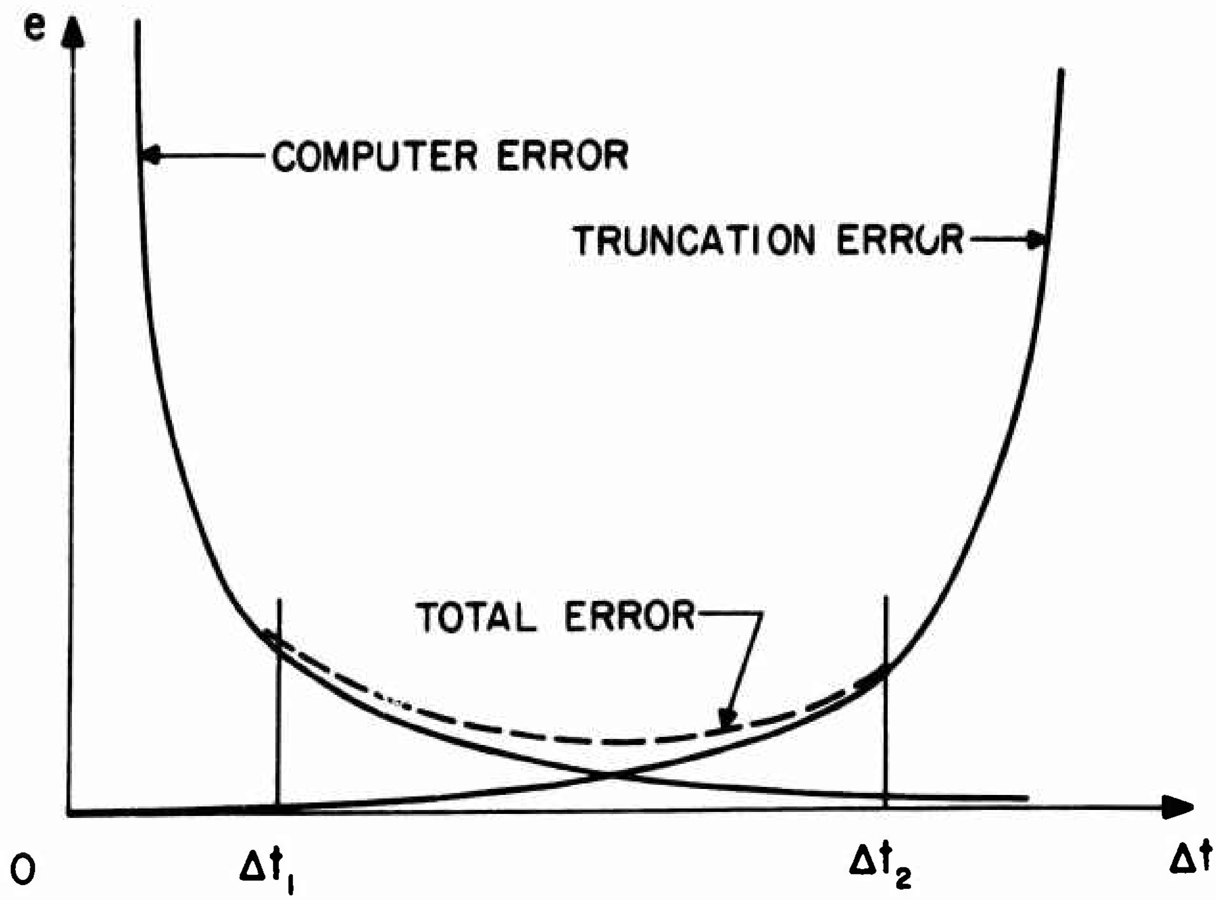

FIG. 14. COMPUTER & TRUNCATION ERRORS

A direct measure of the iteration error is provided by the maximum absolute value of the unbalanced generalized force component at the end of the time step

$$\psi_b = \max|\psi_i(\Delta t)| \quad , \quad i = 1, 2, \ldots, n \tag{3.20}$$

The minimization process is continued until

$$\psi_b \leq \psi_u$$

where ψ_u is a prescribed upper bound on the force imbalance.

SECTION 4

SUMMARY

This report describes the mathematical models and the solution process which form the basis of the computer program SINGER. The function of SINGER is to predict the behavior of plane skeletal reinforced concrete structures in their environments. Of primary interest is the transient nonlinear response including element failures and structural collapse.

The principal features of the mathematical models and the solution process are summarized below:

ACTIONS

Actions, mathematical models of the environment, consist of the self-weight of the structure, distributed and concentrated static and dynamic loads, inertia forces, and support motions. All distributed forces are replaced by equivalent nodal forces. Lumped masses are assigned to the nodal degrees-of-freedom.

SYSTEM MODEL

a. The structure is represented by an assemblage of line elements (models of reinforced concrete beam-columns) and springs (models of partial joint releases) interconnected at a finite number of nodes.

b. The state of the system is characterized by the work function, a scalar function that contains implicitly all the forces acting on the system. The work function is uniquely defined in terms

of the generalized coordinates, which must be related to the equilibrium path (motion) when the system behaves nonlinearly.

c. There is no direct restriction on the magnitude of the generalized coordinates, which consist of nodal-displacements, relative release-displacements, and internal element-displacements. However, relative displacements of nodes linked by elements are limited by the small deformation requirements of the elements. Violations of these limitations can be resolved through subdivision of the elements.

d. The transformation of the large nodal displacements into relative element displacements is expressed with the aid of two frames of reference: The global frame of reference, which is fixed in space, is used to describe nodal properties (e.g., initial state, displacements, forces); the deformation frame of reference, a moving frame of reference, is used to describe element properties (e.g., strains, stresses, distortions).

e. In a static analysis, system failure, structural collapse, is linked to instability of equilibrium. In a dynamic analysis, structural collapse is inferred from the motion of the system.

ELEMENT MODEL

a. The beam-column, the basic structural element, is modeled as a one-dimensional continuum, which is discretized. Axial and flexural deformations are modeled explicitly; only a measure of shear distortions and their significance is provided. Deformations are limited by the assumption that strains and rotations are small relative to unity. Inelastic deformations are modeled up to element

failure. Energy dissipation induced by inelastic behavior accounts for structural damping.

b. The beam-column effect, the coupling of axial and flexural deformations is represented by the corresponding nonlinear term in the strain-displacement relation. The varying neutral axis, a characteristic of beam-columns, is modeled by admitting normal strain variations along the reference axis. This feature makes it also possible to locate the reference axis anywhere in the longitudinal plane of symmetry of the element; thus it eliminates modeling of joint eccentricities.

c. Excessive deformations associated with slender elements or "plastic" hinges are controlled by the division of the element into subelements.

d. Constitutive laws for concrete (unconfined and confined) and reinforcing steel are described in the form of piecewise linear stress-strain curves. Material behavior under monotonic and cyclic loading is modeled.

e. Element failure, which is defined as the limit of continuous change of state, is predicted on the basis of lower-bound criteria. Modifications of these criteria are formulated to permit more probable failure predictions.

RESPONSE

The solution process initiates at a point where the state of the system is completely defined and proceeds along discrete points of the motion: The time function of each generalized coordinate is approximated within two successive points in time, the time step, by a finite

power series whose undetermined coefficient corresponds to the unknown displacement at the end of the time step. This representation of the motion permits one to formulate the work function of the system at the end of the time step in terms of the unknown generalized coordinates. The desired equilibrium state is obtained by minimization of the work function.

LIMITATIONS

Spatial and temporal discretization and the inherent variability of material properties form the principal sources of error. Spatial discretization errors can be controlled through the subdivision of elements. Although internal energy computation is based on a fixed mesh imposed on the longitudinal plane of the element, the reduction of the element length results in a mesh refinement, and, hence it improves the accuracy of the energy computation. Temporal discretization errors can be controlled by varying the size of the time step. The constitutive laws governing material behavior are described by deterministic models, which do not reflect the randomness of some properties such as the fracture strength. Consequently, element failures precipitated by material fracture are monitored via lower-bound criteria.

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NOTATION

A, A_s = area, effective shear area of beam

A, B, C, D, E = transformation matrices

$B, \bar{B}$ = compatibility matrices

c = $\cos \alpha$

c_i = $\cos U_{i3}$

$\bar{d}$ = rigid-body configuration of element in deformation coordinates

e_i = energy imbalance corresponding to i^{th} generalized force component

e_a, e_b = measures of iteration error at the beginning, end of time step Δt

e_{ab} = measure of truncation and iteration errors at $\Delta t/2$

e_T = measure of truncation error

e_ℓ, e_u = lower, upper bound on e_T

E, G = Young's, shear modulus of elasticity

f_a, f_b = element force vectors at the a, b-end

f_{ai}, f_{bi} = element forces at the a, b-end

f_b, f_b^e = generalized applied, equivalent force vector at the end of time step Δt

h = height of beam

I = moment of inertia of beam

I = identity matrix

k_o, k_1, k_i = stiffness coefficients

K_T = tangent stiffness matrix

L = length of beam

m = diagonal mass matrix

n = number of generalized coordinates

p, p_b = generalized external force vector, at the end of time step Δt

Δp = unbalanced force vector (scalar)

r_b = generalized internal force vector at the end of time step Δt

s = $\sin \alpha$

s_i = $\sin U_{i3}$

s_{i2} = $\sin (U_{i3}/2)$

t = time

t_1, t_2, t_i = specific values of t

Δt = time step

u, v = deflections of point $(x,0)$ on the reference axis

$\bar{u}$, $\bar{u}_i$ = element distortion vector, component

U = internal energy

δU = 1st variation of internal energy

U_s = internal energy induced by shear deformations

U_i = displacement vector of joint i in global coordinates

U_{i3} = rotation of joint i about 3-global axis

$\Delta U = U_j - U_i$ = relative joint displacement vector in global coordinates

U^* = internal-energy density

U_d^*, U_r^* = dissipative, recoverable internal-energy density

U_1^*, U_2^* = internal-energy density at time t_1, t_2

U^*_{12} = change in internal-energy density during the time interval t_1, t_2

V = shear force

V = volume

W = work function

δW = total virtual work

δW_e = external virtual work

x,y = element deformation axes

x,x_i = generalized coordinate vector, component

x_n = n^{th} trial solution

Δx_n = correction to x_n

$x_b,\ddot{x}_b$ = generalized displacement, acceleration vector at the end of time step Δt

$x_{ai},\dot{x}_{ai},\ddot{x}_{ai}$ = displacement, velocity, acceleration at the beginning of time step Δt

x_{bi} = displacement at the end of time step Δt

δx_b = virtual displacement vector at the end of time step Δt

X_i,X_j = position vectors of joints i, j in global coordinates

X_{ij} = global coordinate of joint i in j direction

$\Delta X=X_j-X_i$ = relative position vector in the initial state

$\Delta X_1,\Delta X_2$ = components of ΔX

$\Delta X^*=\Delta X+\Delta U$ = relative position vector in the displaced state

α = angle between 1-local axis of element and 1-global axis

β_i = coefficient

γ = shear deformation factor

δ = virtual variation

$\varepsilon_o, \varepsilon$ = normal strains

ε_i = normal strain at time t_i

η, ξ = nondimensional element deformation axes

κ = shape factor

σ = normal stress

ϕ_i = element shape function

ϕ_i' = $\dfrac{d\phi_i}{d\xi}$

ϕ_i'' = $\dfrac{d^2\phi_i}{d\xi^2}$

ψ_b = measure of iteration error at the end of time step Δt

ψ_i = i^{th} unbalanced generalized force

ψ_u = upper bound on ψ_b

APPENDIX A

CONSTITUTIVE LAWS FOR CONCRETE AND STEEL

This appendix summarizes the uniaxial stress-strain curves used in describing the material response of a single fiber of either concrete or steel. The curves described are the default stress-strain curves generated by the program. The user has the option to specify others if he so desires.

A.1 ASSUMPTIONS ON MATERIAL BEHAVIOR

The following assumptions have been made in developing the constitutive models presented herein:

1. Stresses in the concrete and steel are uniquely related to the strains. For direct tension and compression tests under short time loading, this is correct. This permits the calculation of the stresses in the concrete and steel once the strains are known.
2. The stress-strain relationship for compressed concrete not confined by lateral reinforcement is identical to that for concrete in direct compression. The neglect of a strain gradient effect in the compression zone of a beam is justified by adequate correlation between experimental results and many flexural theories based on this assumption.
3. The stress-strain relationship for compressed concrete confined by lateral reinforcement has a strength greater than the unconfined direct compression strength. Data are presented in section A.2 which supports and quantifies this assumption.

4. Tension stress in concrete is neglected. The magnitude of the tensile stresses in the concrete is small compared to that in the reinforcement and their neglect will not significantly change the results of the analysis.
5. Concrete stress-strain curves are valid for normal weight concrete and compressive strengths between 2500 psi and 8000 psi. Lightweight and heavyweight concretes are excluded. There is sufficient test data to generalize the curves presented to only a limited range of concrete strengths.
6. The stress-strain relationships for steel can be determined from tension tests for both the behavior in tension and compression. Complete stress-strain curves, including strain hardening and breaking strengths, are given in section A.3. These are limited to steels with yield points from 33 ksi to 75 ksi.
7. Creep, shrinkage, and temperature effects are ignored. For short duration loadings, the first two effects can be neglected. The change of material properties with temperature is not sufficiently documented for reinforced concrete and therefore is omitted.
8. Strain rate effects on material response are neglected. This can influence the stress-strain response at local points in the structure. However, it is assumed that the overall response of the structure will not be significantly affected by ignoring this complexity.
9. Adequate lateral support is present to prevent buckling of steel in compression. This assumption is valid as long as the concrete cover is intact. After spalling has taken place, lateral support must be provided by lateral ties or stirrups.

In developing the computer code, checks are made on the above assumptions whenever possible. For example, the concrete compressive strength given must be within the range specified, stirrup spacing is checked against a requirement for prevention of local buckling, etc. If one of the assumptions is violated, a warning is given to the user that he is using the program beyond its intended application.

A.2 STRESS-STRAIN RELATIONSHIP FOR CONCRETE

If concrete is compressed in one direction, it tends to expand laterally. If this expansion occurs freely, the concrete is said to be "unconfined" and principal compressive stresses exist in one direction only. On the other hand, if such lateral expansion is restricted, the concrete is said to be "confined" and, as a result of such restriction, compressive stresses develop in all directions. Up to the stage corresponding to crushing the behavior of the concrete is essentially that of the unconfined concrete. Beyond this stage, the concrete core bound by lateral reinforcement has greater strength and ductility than the unconfined concrete. Because of these differences, it is necessary to describe stress-strain relationships for both types of concrete.

A.2.1 Stress-Strain Curve for Unconfined Concrete

The default stress-strain curve for unconfined concrete is given in Figure A.1. A non-dimensional plot is not possible because the slope of the descending branch is dependent on the compressive cylinder strength, f'_c. The curve is divided into two portions, AB and BC. For the region AB, a parabolic expression (represented by a series of straight line segments in the program) given in reference A.1 is used:

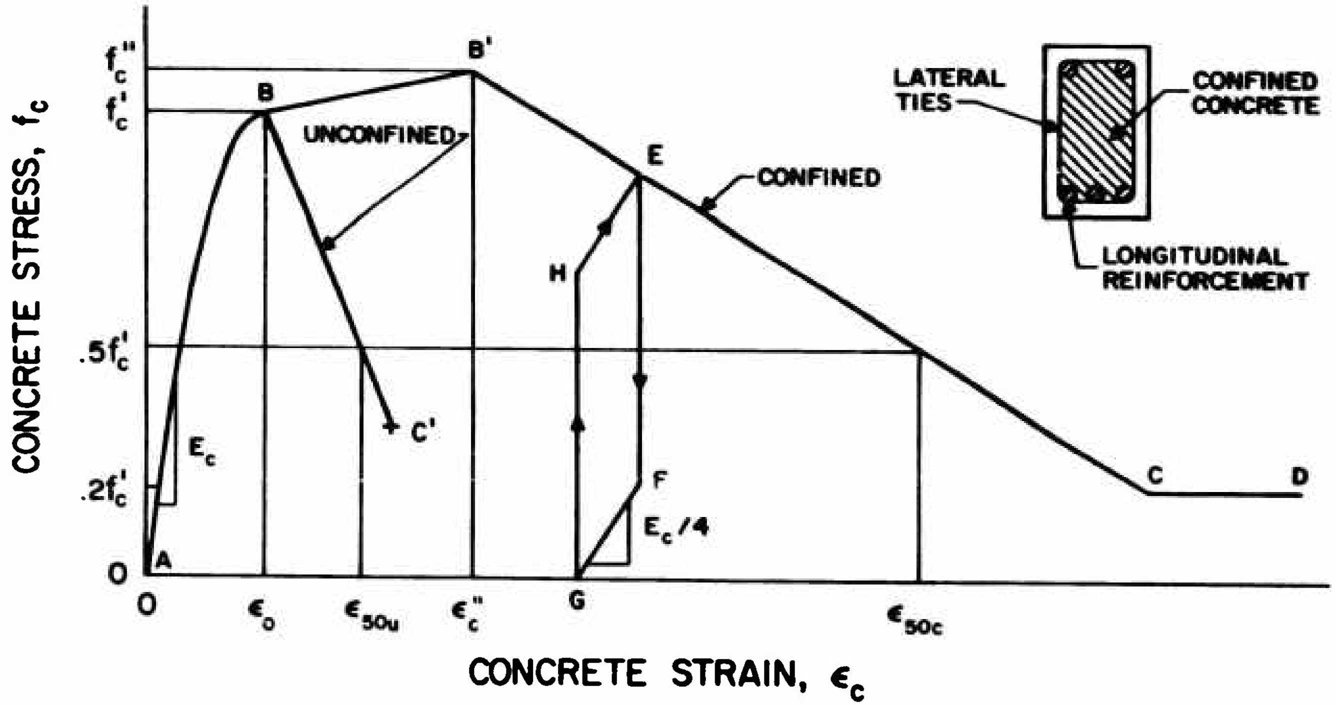

FIG. A. 1: COMPLETE STRESS-STRAIN CURVE FOR CONCRETE

$$f_c = f'_c \left[\frac{2\varepsilon_c}{\varepsilon_o} - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2 \right] \qquad \text{(A.1)}$$

in which the strain at maximum stress is assumed to be $\varepsilon_o = 0.002$. It is also assumed that the maximum stress is the cylinder strength f'_c, i.e., the factor 0.85 is not included. The reason for this is that the 0.85 factor was based on column tests without a strain gradient. When a strain gradient is present, such as in a member in bending, observations have shown (reference A.2) that a factor of 1.0 is conservative. The region BC' is defined by a straight line whose slope is determined by the strain ε_{50u}, when the concrete stress has fallen to 50% of the cylinder strength of the unconfined concrete. This is given in reference A.3 as

$$\varepsilon_{50u} = \frac{3 + 0.002f'_c}{f'_c - 1{,}000} \qquad \text{(A.2)}$$

in which f'_c is expressed in pounds per square inch. The straight line is continued until the concrete strain reaches the failure value defined in Appendix B, section B.3.1. At this point the unconfined concrete is no longer effective and is removed from the cross-section.

A.2.2 Stress-Strain Curve for Confined Concrete

The default stress-strain curve for confined concrete is also given in Figure A.1. The curve is divided into four regions and is similar to the curve given in reference A.4. For the region AB, the curve is identical to that given by Eq. A.1.

For the region BB', a modification in the curve of reference A.4 is made to include an increase in compressive strength when lateral ties are present. Recommended increases for this region vary from nearly 50% (reference

A.5) to zero (reference A.6). However, a majority of the researchers indicate that a modest increase is reasonable, and the following expression is used:

$$f''_c = f'_c + \Delta f_c \tag{A.3}$$

in which f''_c = confined concrete compressive strength and Δf_c = increase in compressive strength over the unconfined value. The magnitude of $\Delta f'_c$ is dependent on the confining action of the transverse reinforcement. A theoretical discussion in reference A.5 indicates that the lateral pressure induced is proportional to $p''f''_s$, where p'' = ratio of the volume of lateral reinforcement to the volume of confined concrete and f''_s = unit stress in transverse reinforcement (which is assumed to be equal to the yield stress).

In Figure A.2 are shown the results of tests on rectangular prisms under concentric load from reference A.7 and the modification recommended in reference A.5 for members in flexure. The higher values in each of the concentric load tests are for high strength concretes, the lower values for medium strength concretes. The limits on the bending results are given to show the trend when only a portion of the confined depth of the section, d'', is in compression (c = depth to the neutral axis). A conservative estimate of the increase in strength is given by the straight line whose equation is:

$$\Delta f_c = \frac{3}{4} p''f''_y \leq 2000 \text{ psi} \tag{A.4}$$

The upper limit is necessary because of the limited range of the test data.

Corresponding to the increased maximum compressive strength is a confined concrete strain, ε''_c, which can be expressed as

$$\varepsilon''_c = \varepsilon_0 + \Delta\varepsilon_c \tag{A.5}$$

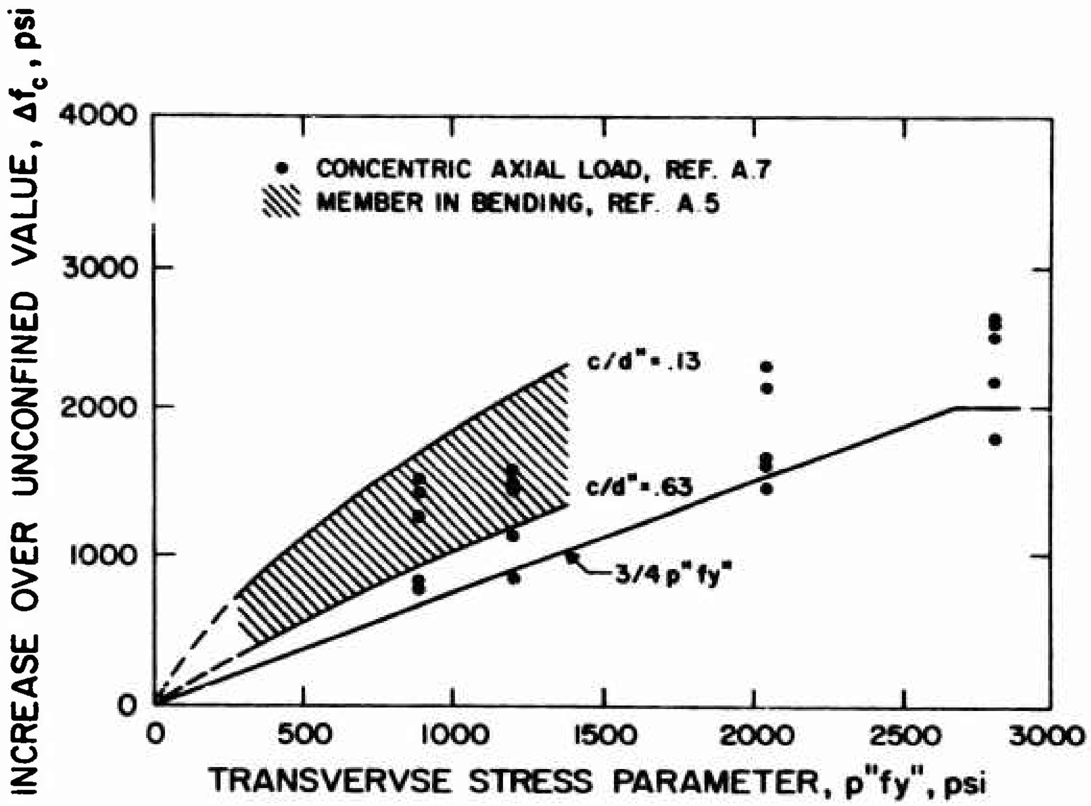

FIG. A.2: ULTIMATE STRENGTH FOR CONFINED CONCRETE

where $\Delta\varepsilon_c$ = increase in strain at maximum stress over the unconfined value. In addition to the volumetric ratio, p", reference A.3 indicates that confined concrete strain is dependent on the ratio of the minimum dimension of the confined core to the spacing of the transverse reinforcement, b"/s.

Experimental results from the tests in reference A.8 and the recommended values of reference A.5 are shown in Figure A.3 for the increase in confined concrete strain. Because of the scatter in the data a lower bound straight line given by the following expression is used:

$$\Delta\varepsilon_c = 0.17p'' \sqrt{b''/s} \leq 0.008 \qquad \text{(A.6)}$$

The upper limit corresponds to a total strain before reaching the descending branch of 0.01.

For the descending branch B'C, the slope is established by the strain, ε_{50C}, at $0.5f'_c$, for the confined concrete and is given in reference A.4 as

$$\varepsilon_{50C} = \frac{3 + 0.002f'_c}{f'_c - 1{,}000} + \frac{3}{4} p'' \sqrt{b''/s} \qquad \text{(A.7)}$$

The first term on the right hand side is identical to Eq. A.2 , thus the second term represents the increase in the 50% strain for the confined concrete over the value for unconfined concrete. The point C on the descending branch is determined by extending a straight line from B' through the 50% point until the concrete stress has fallen to 20% of f'_c.

For the region CD, it is assumed that the concrete can sustain a stress of $0.2f'_c$ for indefinitely large strains. This has been assumed previously in the analysis used in reference A.3 and member failure occured (fracture of tensile steel) before the concrete strains became unrealistic.

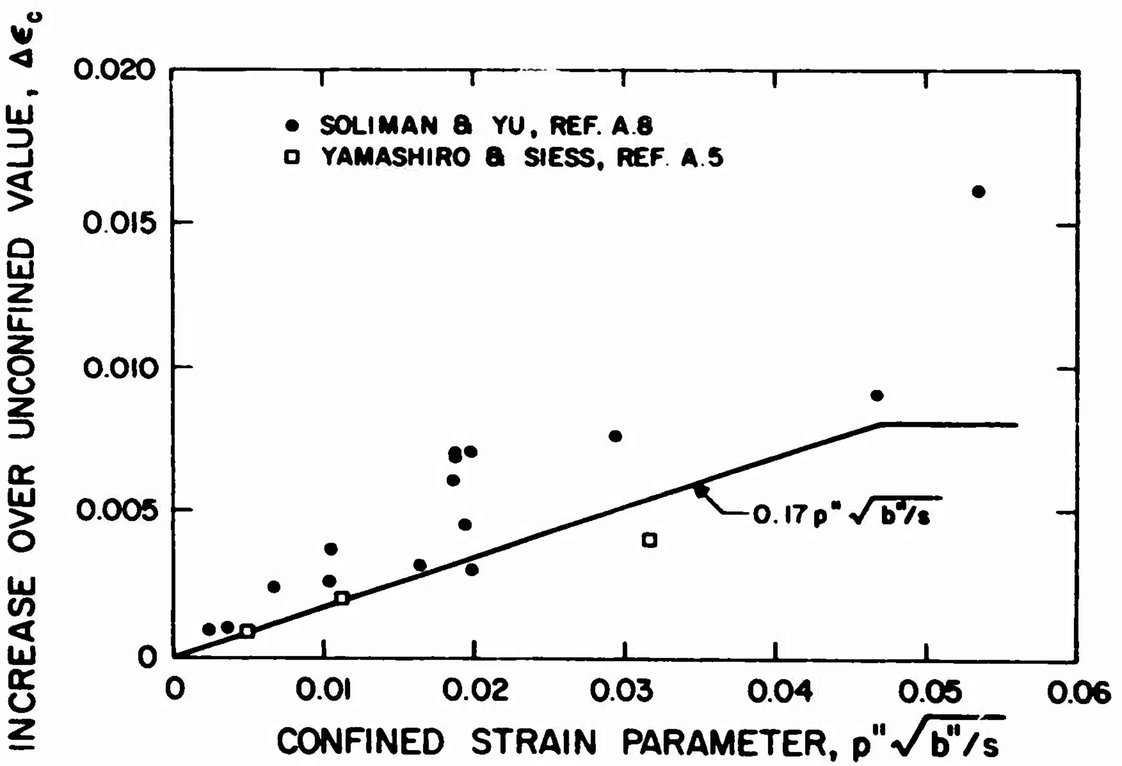

FIG. A.3: STRAIN AT ULTIMATE STRENGTH FOR CONFINED CONCRETE

A.2.3 Cyclic Loading Response of Concrete

The behavior of concrete under repeated loading is also shown in Figure A.1. Unloading and reloading that occurs before point B' (or point B in the case of unconfined concrete) is assumed to follow the initial tangent slope E_c. Reversed loading on the descending branch of either the unconfined or confined stress-strain curve is referred to as "drop-elastic." For example, on unloading from point E, it is assumed that 0.75 of the previous stress is lost without a decrease in strain (the "drop" portion) and then a linear path of slope 0.25 E_c is followed to point G (the "elastic" portion). If the concrete continues to unload, the tensile strains increase without any tensile stress developing. On reloading the strain must regain the value at G before compressive stress can be sustained again. Note that the average slope of the assumed loop between E and G is parallel to the initial tangent modulus of the stress-strain curve.

This representation of the cyclic loading behavior is taken from reference A.4. It can be modified by changing the value of the slope from F to G. A user can input the value of the slope as a constant k times the initial tangent modulus. The value of k = 0.25 is the default condition.

A.3 STRESS-STRAIN RELATIONSHIP FOR STEEL

The default stress-strain curves utilized for steel are shown in Figure A.4. These curves cover a yield point, f_y, range from 33 ksi to 75 ksi, and strains from zero to the breaking point. They include two structural steel grades with yield points equal to 33 ksi and 36 ksi. All of the curves have an elastic portion AB with a constant modulus of elasticity, $E_s = 29 \times 10^3$ ksi. The strain at the beginning of yield, ε_y,

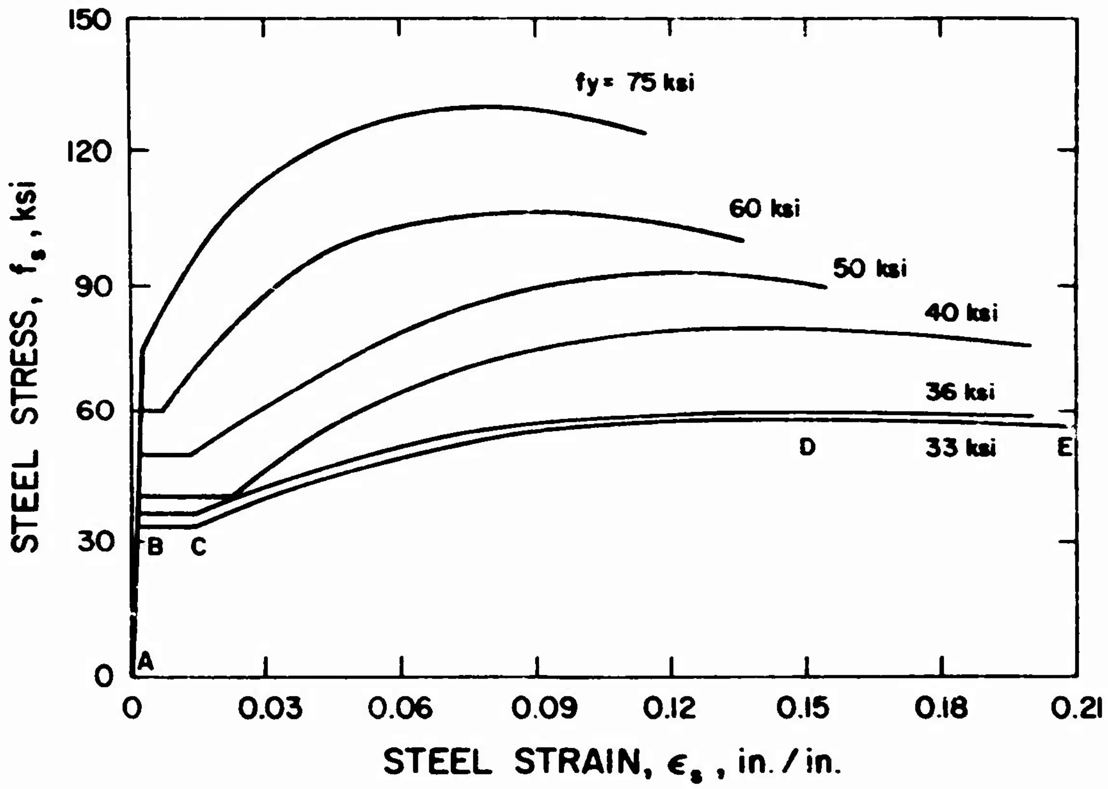

FIG. A.4: TYPICAL STRESS-STRAIN CURVES FOR STEEL

is equal to f_y/E_s. The yield plateau BC varies with the yield strength of the steel. Typical values for the strain at which strain hardening begins, ε_{sh}, are given in Table 1.

The strain hardening curve CDE reaches a maximum stress, f_u, at a strain, ε_u, before dropping off slightly at the breaking strain, ε_b. Typical values of these quantities are also given in Table 1. The following expression for the strain hardening portion (represented by a series of straight line segments in the program) was adapted from one developed in reference A.9

$$f_s = f_y\left[1 + \frac{\varepsilon_s - \varepsilon_{sh}}{\varepsilon_u - \varepsilon_{sh}}\left(\frac{f_u}{f_y} - 1\right)\exp\left(1 - \frac{\varepsilon_s - \varepsilon_{sh}}{\varepsilon_u - \varepsilon_{sh}}\right)\right] \quad (A.8)$$

Table 1

TYPICAL VALUES FOR STEEL STRESS-STRAIN CURVES

f_y, ksi	f_u, ksi	ε_y	ε_{sh}	ε_u	ε_b
33	58	0.00114	0.014	0.15	0.21
36	60	0.00125	0.014	0.15	0.20
40	80	0.00138	0.023	0.14	0.20
50	92	0.00173	0.013	0.12	0.154
60	106	0.00208	0.0060	0.087	0.136
75	130	0.00260	0.0027	0.073	0.115

When the loading is reversed after the yield strain has been reached, the shape of the stress-strain curve is changed because it no longer has a well-defined yield point upon reloading. Figure A.5 shows the general behavior assumed for the steel when reverse loading occurs. On first loading to point 1, the virgin curve described previously is followed. On unloading from point 1 to point 2, the path is parallel to the initital elastic slope. When loading in the opposite direction from point 2 to point 3, the yield point is missing and the curve is described by Eq. A.8 with the origin shifted to point 2. Subsequent cycles of unloading and reloading follow the same pattern and are shown in Figure A.5.

In a previous investigation (reference A.10), a degradation of stiffness with cycles of loading was proposed for the reinforcing steel. However, a study of the original paper (reference A.11) on which the proposal was based revealed that tests were conducted for only one bar size (No. 11) and one yield stress (50 ksi). To extrapolate these results to the general behavior of all bar sizes with a range of yield points from 33 ksi to 75 ksi is not justified. Furthermore, the data of reference A. 11 showed an increase in stiffness in some cycles.

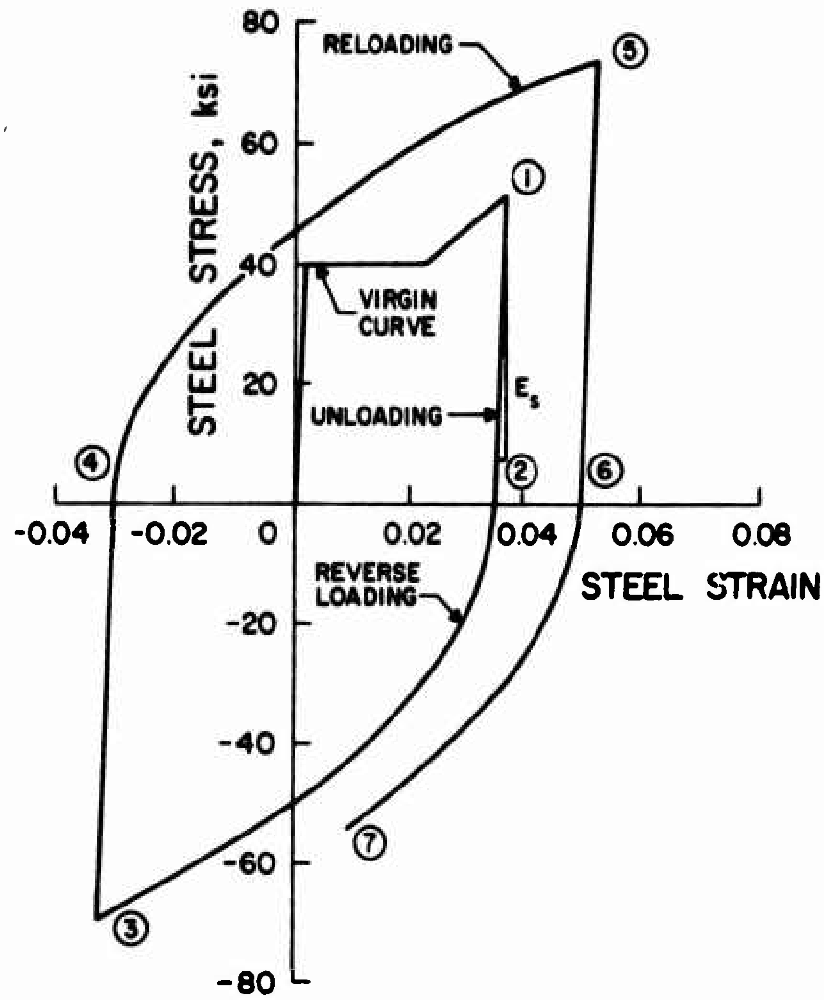

FIG. A. 5: REVERSE LOADING BEHAVIOR OF STEEL

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NOTATION

E_c = Young's modulus of elasticity for concrete

E_s = Young's modulus of elasticity for steel

b'' = breadth of confined concrete cross-section

c = depth to neutral axis from compressive face

d'' = depth of confined concrete cross-section

k = unloading constant for concrete hysteresis loop

p'' = volumetric ratio of transverse reinforcement

s = longitudinal spacing of transverse reinforcement

f_c = compressive stress in concrete

f'_c = compressive strength of 6 by 12 in. cylinders

f''_c = compressive strength of confined concrete

f_s = stress in longitudinal reinforcement

f''_s = stress in lateral reinforcement

f_u = maximum steel stress in strain hardening region

f_y = yield stress of longitudinal reinforcement

f''_y = yield stress of lateral reinforcement

Δf_c = increase of concrete strength over unconfined value

ε_b = breaking strain of longitudinal reinforcement

ε_c = compressive strain in concrete

ε''_c = strain in confined concrete at maximum stress

ε_o = strain in unconfined concrete at maximum stress

ε_s = strain in longitudinal reinforcement

ε_{sh} = steel strain at onset of strain hardening

ε_u = strain corresponding to maximum steel stress

ε_y = yield strain of longitudinal reinforcement

ε_{50C} = confined concrete strain on falling branch at $0.5f'_c$

ε_{50u} = unconfined concrete strain on falling branch at $0.5f'_c$

$\Delta\varepsilon_c$ = increase of concrete strain over unconfined value

APPENDIX B

ELEMENT FAILURE CRITERIA

The behavior of the structural system is dependent upon the behavior of each component element since each one contributes to the total energy of the system. The process of failure is also related to the failure of the individual elements. Some provision must be made to define and predict failure in an element. This is the purpose of the element failure criteria.

B.1 DEFINITIONS

Failure of a real concrete member can be associated with an abrupt loss in its ability to resist applied loads. Failure is caused by localized behavior, and it is usually associated with fracture of material.

Failure of the element model is defined as those states which correspond to the physical failure modes of a real member and determine a limit to the continuum model behavior. These states are detected by assigning specific values to certain quantitites which can be related to variables in the mathematical model. Since these variables are directly associated with physical behavior, their values must be obtained from test results.

The derived expressions which relate these numerical quantities to the corresponding quantities determined from the element model are referred to as the element failure criteria. Each failure criterion is developed and applied consistent with the mathematical model of the physical system and the actions.

B.2 CLASSIFICATION OF FAILURE MODES

A failure mode is a distinct manifestation of failure. The prediction of each specific failure mode is made with the corresponding failure criteria. Some criteria can be directly related to the continuum strain

or stress states at a point, while others rely on an indirect measure of strength through stress resultants and element properties. The former type of criteria are referred to as "micro criteria"; the latter are called "macro criteria".

The classification of failure is made according to the dominant stress state within an element at critical sections defined for each failure mode; (critical sections are discussed in section B.4.2). A typical element with stress resultant variation along the length is shown in Fig. B.1. Accordingly, dominant stress states may be associated with either the bending moment, the shear force, or the axial force. Since normal strain and stress values are defined at each point by the continuum model, the dominant normal stress effects (flexural failure and axial force failure) are predicted by micro criteria based on limiting strain or stress values. The shear stresses are not predicted in a direct way; a nominal (average) shear stress distribution can be measured in an indirect way based on equilibrium requirements for the gross element. Therefore, the shear-flexure failure is detected by a macro criteria.

The three basic failure categories are:

1. Flexural failure: dominant normal stress state caused by bending, (micro);
2. Shear-flexure failure: dominant shear stress effect in addition to the normal bending stress caused by a variation in bending moment (macro); and
3. Axial force failure: dominant normal stress state caused by a large axial force, (micro).

Since all three stress resultants (f_{x1}, f_{x2}, f_{x3}) can be associated with any of the three categories of failure, the distinction between case 3

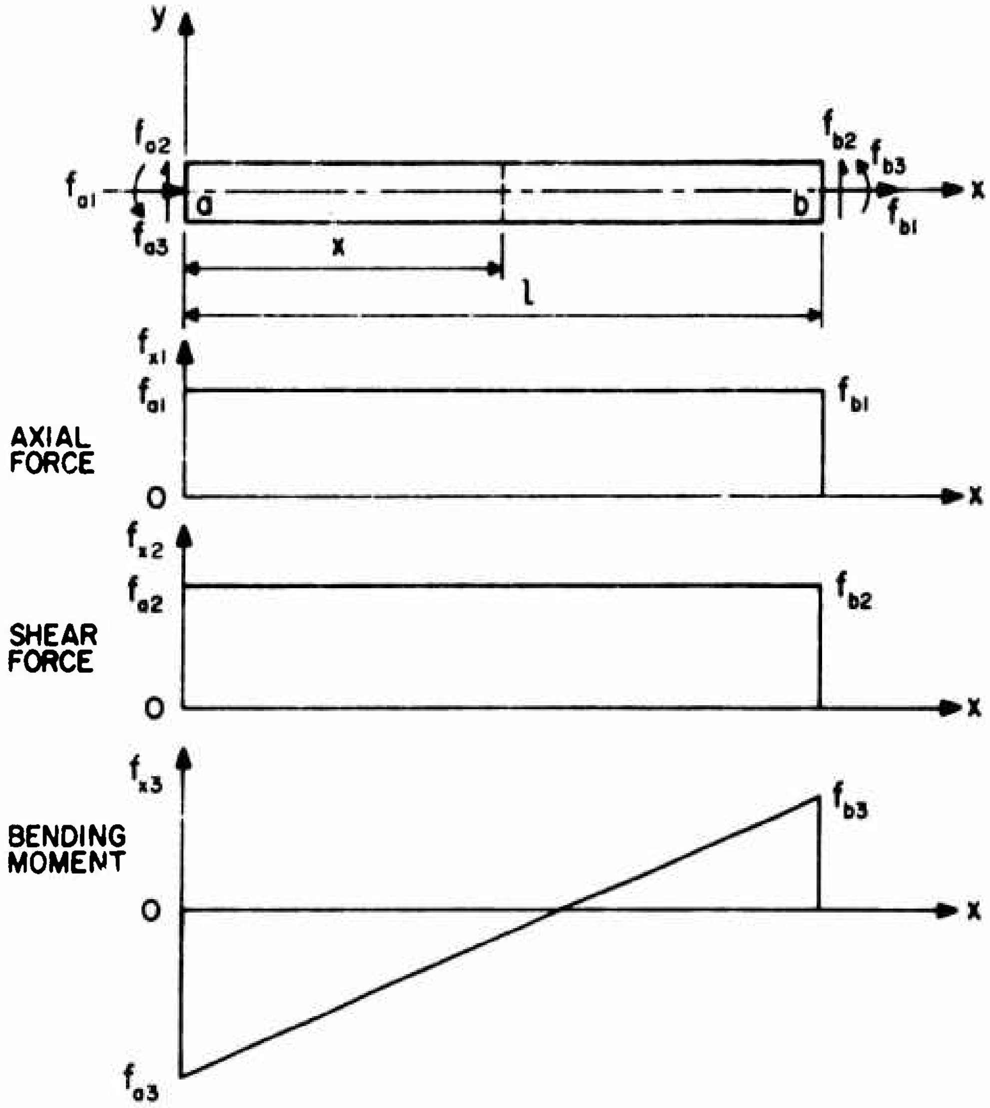


FIG. B. 1: ELEMENT STRESS RESULTANTS

and the other two is made on the basis of the strain state at the critical sections; cases 1 and 2 are associated with strain states which have a point of zero strain within the dimensions of the cross section; case 3 corresponds to the condition of tension or compression across the entire section, since the zero strain point falls outside the section dimensions.

Each failure category can be further classified according to the possible failure modes. The set of failure modes for each category is determined by the type of reinforcement and the type of failure possible for the dominant stress conditions prescribed. The sets of failure modes for the failure categories are defined in the flow charts of Figs. B.2,3, and 4. The failure criteria corresponding to the failure modes are developed in section B.3.

B.3 FAILURE CRITERIA

Each failure mode defined in section B.2 requires a failure criterion. In addition to measuring the limiting condition for the dominant effect, the criterion must include other effects characteristic of possible system behavior. This includes: secondary stress effects, which provide any alterations to the basic criteria caused by stresses other than the dominant stress; and the effects of previous loading history, which measure any change to the basic form caused by the stress variation experienced in an element during previous load conditions, such as load reversals. In addition, a user modification capability is incorporated so that the magnitudes of the basic criteria may be changed to allow certain effects to be studied. The secondary stress effects and the loading history effects are physical measurements obtained from published test results. The user modification capability is provided through

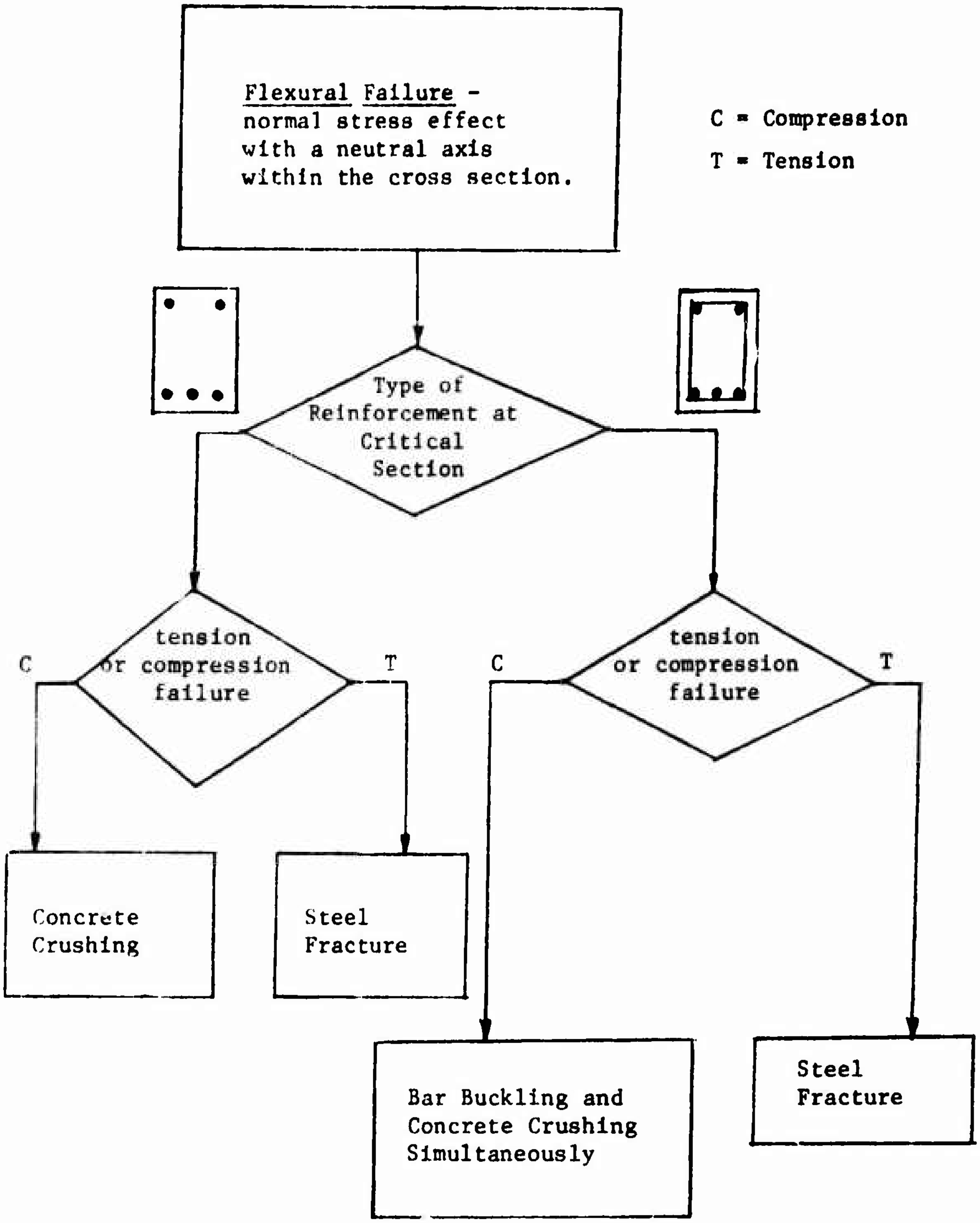


Fig. B.2: Flexural Failure Classification

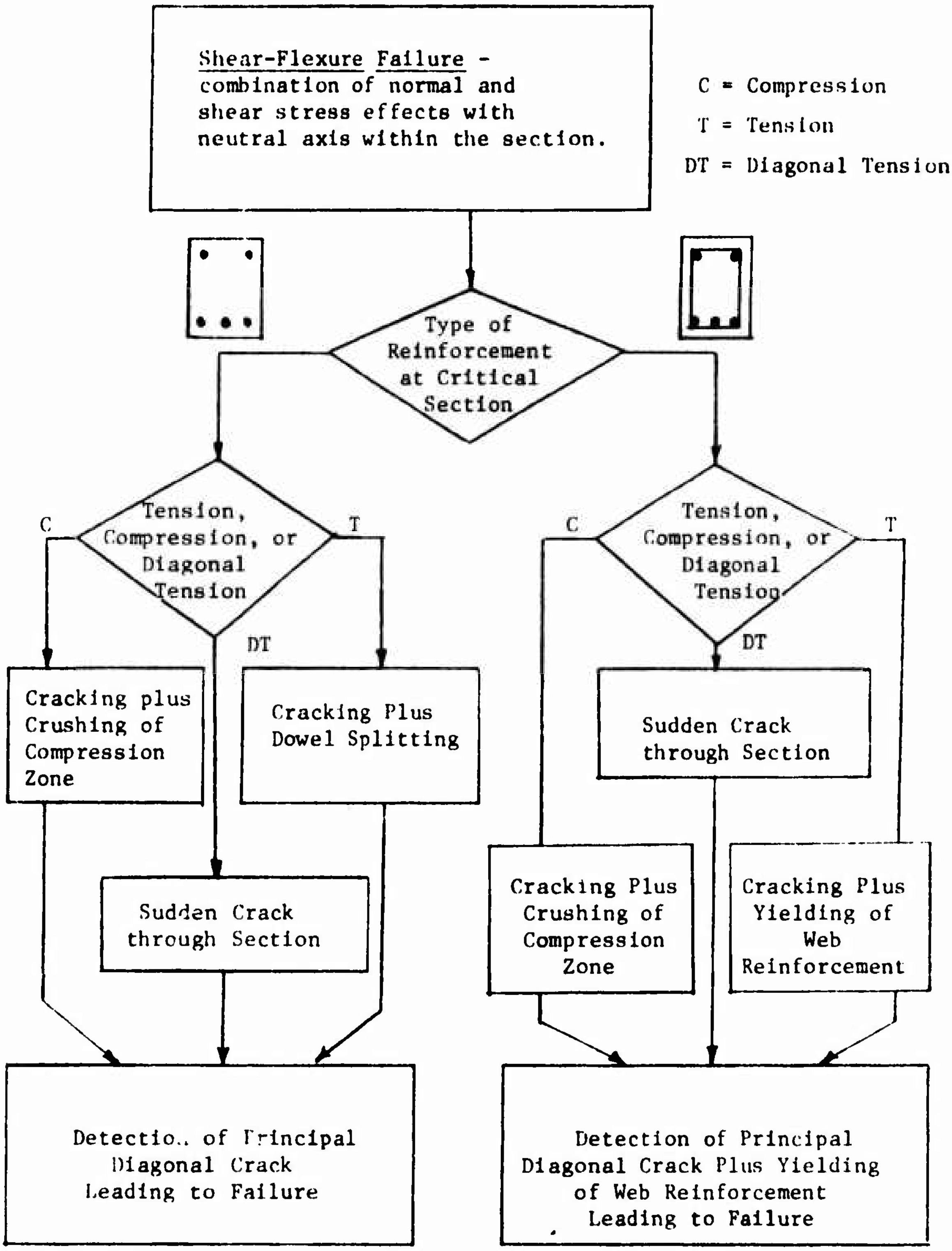


Fig. B.3: Shear-Flexure Failure Classification

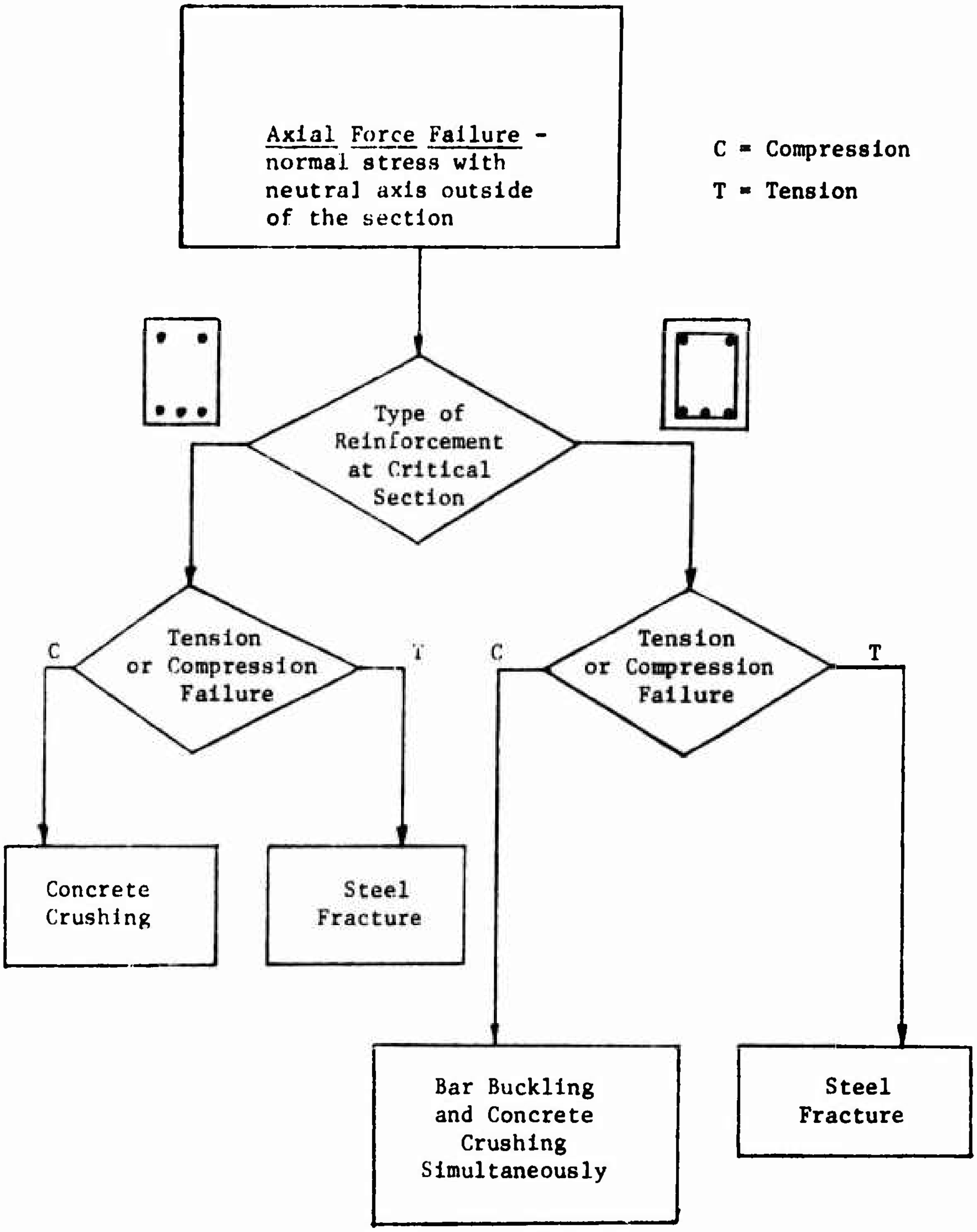


Fig. B.4: Axial Force Failure Classification

coefficients built into the criteria with prescribed limits of variation.

All of the test data used in the development of the failure criteria were obtained from published sources. Not all of the specific information required was available; the deficiencies encountered are noted in the criteria developed.

Since the failure being measured was usually associated with some form of fracture, the test data contained a variable amount of uncertainty indicated by some degree of scatter in the plotted form. To represent these results in the form of a deterministic expression, a reasonable lower bound function was chosen in each case.

The individual failure criterion is developed according to the following outline:

1. Basic criterion for dominant stress,
2. Effect of secondary stresses,
3. Effect of loading history,
4. User modification and
5. Assumptions.

The relationship in each case has the general form expressed by:

$$\boxed{\text{(Specific criterion value)} - \text{(computed model value.)} \lesseqq 0}\ .$$

The individual expressions are also put in a dimensionless form so that units are not involved in their application. The one exception to this rule is the strain criterion for concrete crushing in B.3.1.1.; the constants in this expression are not dimensionless, even though the total expression is dimensionless. All modification coefficients are dimensionless.

The essential details of all of the failure criteria are summarized in section B.3.4.

B.3.1 FLEXURAL FAILURE

This failure category is specified by a dominant normal stress state at a section in an element caused by bending. Another distinction is that the section has a point of zero strain within the dimensions of the cross section. The failure modes which require a failure criteria are shown in Figure B.2.

B.3.1.1 CONCRETE CRUSHING

This failure mode is the crushing of the concrete in an unconfined compression zone. The concrete crushing may occur progressively from the outside surface inward, or it may occur in a sudden disintegration of a highly stressed region. Since the concrete compressive stress-strain response has a negative slope beyond an ultimate stress point, (see Appendix A, Fig. A.1.), a sufficiently large curvature can cause a significant region of the compression zone to be within the negative slope influence. At some point during an increasing load, the bending moment resistance at the section reaches a peak value. If there are no other regions which can carry any additional load increment, then the section will disintegrate suddenly. This condition is characteristic of a singly reinforced beam when a compression failure occurs before the steel yields in tension, [B.34]. A discussion of this mode of behavior is given in references B.13, 18, 19 .

Before the disintegration state is reached, the outer concrete may begin to crush locally. If this happens, part of the moment resisting

capacity is lost at that section. The uniform cross section property is lost, and a stress concentration effect is created in the element. In addition, the spalling may be irregular causing a loss in symmetry with respect to the plane of the structure loading.

If there are steel bars in the compression zone of the concrete, crushing can still occur, but at a proportionally larger moment due to the load carried by the steel. After the crushing begins, the behavior of the steel bars in compression is uncertain without web reinforcement to contain them.

The entire nonlinear response up to an ultimate state can be predicted by the element model since the complete stress-strain curves are included. However, after crushing of the outer layers of concrete, the related effects on the element cannot be predicted by the model. Consequently, the limit to the continuum model behavior is associated with the concrete crushing state at a critical section. The criterion developed is assumed to be valid for a compression zone with or without compression steel.

The condition of initial crushing can be defined by a maximum strain value for unconfined concrete. The stress-strain curves in Figure B.7 show that the maximum strain values decrease with increasing concrete strength f'_c. This characteristic is reflected in both ε_{50u} and ε_{20u} strain points for unconfined concrete defined in appendix A, Fig. A.1, and by the equations:

$$\varepsilon_{50u} = \frac{3.+0.002f'_c}{f'_c-1000} \tag{B.1}$$

and $$\varepsilon_{20u} = 1.8572\ \varepsilon_{50u} - 0.8572\ \varepsilon_o \tag{B.2}$$

where f'_c = ultimate concrete cylinder strength

ε_o = strain at ultimate stress = 0.002

A comparison of these two strain points are shown in Figs. B.5 and B.6 as a function of the variable f'_c. On the same figures are experimental values of ultimate strain due to flexure. It appears that ε_{50u} is a realistic lower bound to the data for the higher strength values of f'_c (f'_c > 4000.psi). For the lower values of f'_c, ε_{50u} is too large; a cut-off value for the ultimate strain at 0.0035 in/in is defined as a lower bound to all of the remaining data points.

In tests of reinforced concrete beams in reference B.31 , flexural compressive strains on the outer surface of highly stressed regions reached the level of 0.004 in/in. before crushing. The f'_c values were in the range of 4000 - 6000 psi.

(1) Basic failure criterion: defined by ultimate strain for concrete in compression:

$$\varepsilon_{f1} - \varepsilon_c \leq 0 \tag{B.3}$$

where $\varepsilon_{f1} = \dfrac{3 + 0.002f'_c}{f'_c - 1000} \leq 0.0035$

ε_c = concrete compressive strain at critical section.

The function ε_{f1} is shown as the lower limit curve on Figs. 5 and 6.

(2) Axial force effect: included in the strain state.

(3) Loading history effect: no measurable difference for a few unloading-reloading cycles, [24, 31, 33]. Basic stress-strain response function forms an envelope to the reloading paths. Reversal of load produces tension which does not affect concrete compression strength.

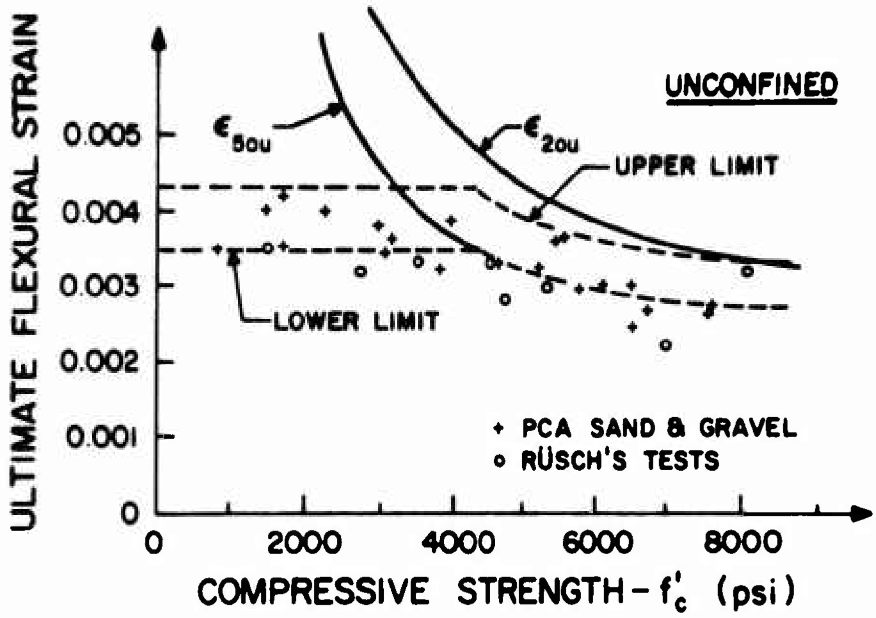


FIG. B.5: ULTIMATE STRENGTH PROPERTIES OF STRESS DISTRIBUTION (DATA FROM REF. [B.15]).

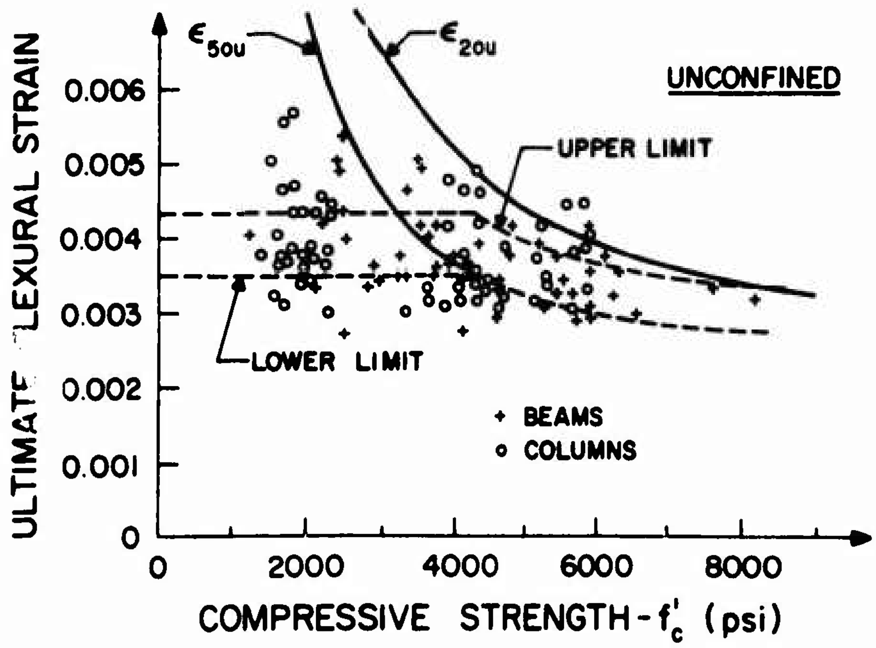


FIG. B.6: ULTIMATE STRAIN FROM TESTS OF REINFORCED CONCRETE MEMBERS (DATA FROM REF. [B.15]).

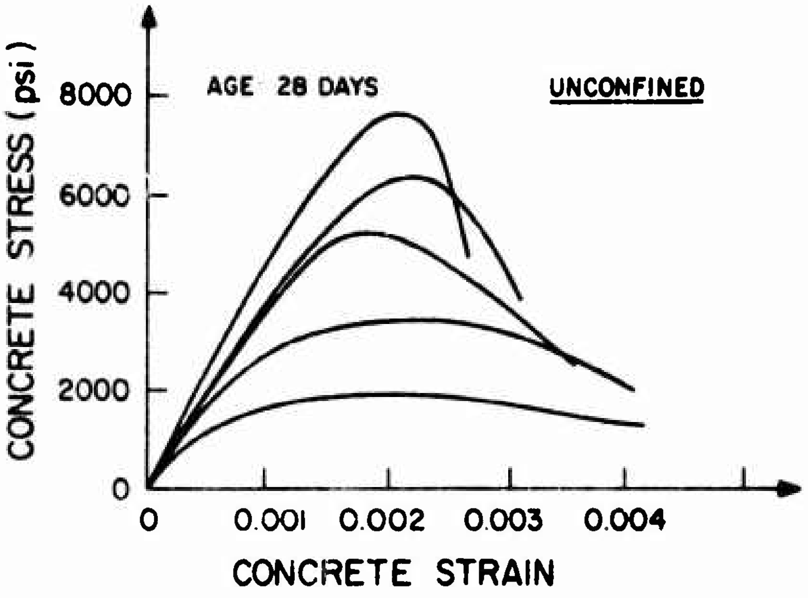


FIG. B.7: FLEXURAL STRESS-STRAIN CURVES (DATA FROM REF. [B.15]).

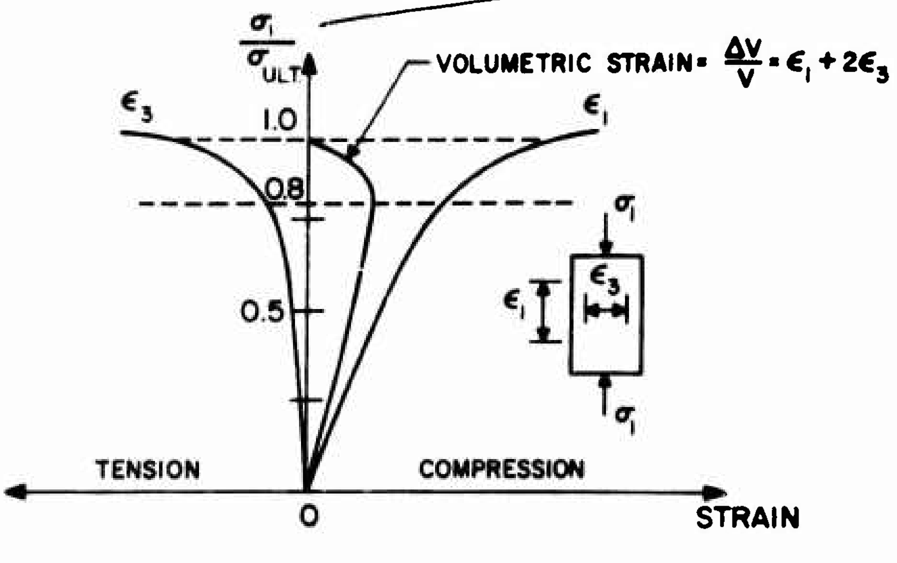


FIG. B.8: CONCRETE STRAIN FUNCTIONS (DATA FROM REF. [B. 28]).

(4) Modification:

(a) modification by coefficient:

$$\varepsilon^*_{f1} = C_1 \cdot \frac{3 + 0.002\, f'_c}{f'_c - 1000} \leq (0.0035) \qquad \text{(B.4)}$$

$$(1. \leq C_1 \leq 1.23)$$

The modified ultimate strain is defined by the relationship: C_1x(lower limit function). The upper limit function is defined by $C_1 = 1.23$. (The upper limit for C_1 is computed on the basis of the evaluation of ε^*_{f1} corresponding to ε_{20u} at f'_c (maximum) = 8000 psi.) The upper limit function is also shown of Figs. B.5 and B.6.

(b) Complete override is not possible since there must be a limit to the strain in unconfined concrete as specified in the material input function. The suggested upper limit for failure is defined above.

(5) Assumptions:

The concrete crushing criterion applies to members with or without compression steel.

B.3.1.2 STEEL FRACTURE

It is possible for the tensile reinforcement to fracture at a critical section before any other limit state is reached for the element. Consequently the fracture of the longitudinal steel caused by excessive tensile strain is considered to be a failure mode.

The occurrence of a bar fracture in a member causes significant changes in the behavior: it creates a severe stress concentration in

the local region due to the lack of material continuity; this in turn causes additional cracking in the concrete and shifts additional stress to the remaining steel bars and the concrete in compression; if there are no other bars at the section, the fracture causes a complete discontinuity in the member. In addition, the stress distribution in the fractured bar varies considerably from zero at the break point to some tensile value consistent with the unbroken bars at some distance away.

For a two-dimensional model, all of the bars at the same distance from the reference axis have the same strain value; and since they are assumed to have the same material properties, the entire row fractures at the limit strain. The model can predict the strain value in the nonlinear range up to the limiting strain value. After the fracture point is reached, the model cannot predict the stress concentration effects or the stress distribution in the fractured bar. Therefore, the limit state is defined as the tensile fracture of any row of the steel reinforcing bars at the critical section of an element.

The failure condition can be predicted on the basis of a strain value at the point on the cross section corresponding to the bar location. The element model defines this value directly. Since the strain is assumed to be uniform across the bar area, the limit strain of the uniaxial stress strain function for steel determines this value. The numerical values for the limit strains are shown in appendix A, Fig. A.4.

(1) Basic failure criterion: defined by a limiting strain value for a steel bar in tension. (See appendix A, Fig. A.4)

$$\varepsilon_{f2} - \varepsilon_s \leq 0 \qquad \text{(B.5)}$$

where ε_{f2} = limiting tensile strain value

ε_s = tensile strain in longitudinal steel bar at a critical section,

(2) Axial force effect: included in the strain state.

(3) Loading history effect: it appears that no information is available on the question of the effect of a few (2 or 3) inelastic stress reversals on the fracture strain of reinforcing steel. The behavior of reinforcing steel subjected to stress reversal is documented in reference B.32 . If the number of stress reversals is small, simplifications may be introduced into the stress-strain function which includes stress-reversal, basing the stress reversal curve on the original monotonic stress-strain response. Others have utilized this concept to define an idealized response for steel with stress reversal, [B.1,8,29]. But none of these have indicated the fracture strength. For simplification, it is assumed that the strain at fracture remains the same as the monotonic fracture point, regardless of the history of loading.

(4) Modification:

(a) no parameter modification is necessary since the fracture point defined by the input function for the material is not altered in the failure criteria

(b) no override is possible because the physical limit of the stress-strain function for each material is defined independent of the limit conditions for the model.

(5) Assumptions:

The limiting tensile strain value for the monotonic stress-strain function for steel is a valid measure of the fracture strength including loading history effects.

Note:

No special check is made for the case where bending in a singly reinforced concrete element occurs opposite to its reinforced strength. Clearly the concrete will crack in tension and form a discontinuity in the members at the section. Physically, the longitudinal bars could be positioned at any depth in a member. Whether or not there is sufficient resisting strength in the couple that is formed by the resulting concrete compression zone and the steel in tension depends on the moment to be resisted and the steel location at a section. If the steel is near the tension surface, a larger moment can be resisted than for the case where the steel is near the compression surface. In any case, the limitations provided by the concrete crushing (B.3.1.1) and steel fracture (B.3.1.2) are sufficient failure checks for any placement of the longitudinal steel.

B.3.1.3 BAR BUCKLING AND CONCRETE CRUSHING SIMULTANEOUSLY

This failure mode is the process of the bending out of reinforcing bars and the simultaneous crushing of concrete causing a sudden destruction of the compression zone. It is characteristic of a member with web reinforcement (see Fig. B.9).

This behavior has several distinguishing traits:

a. The condition is not possible until the concrete outside of the compression bars has begun to crush; otherwise the bars are adequately restrained from buckling;

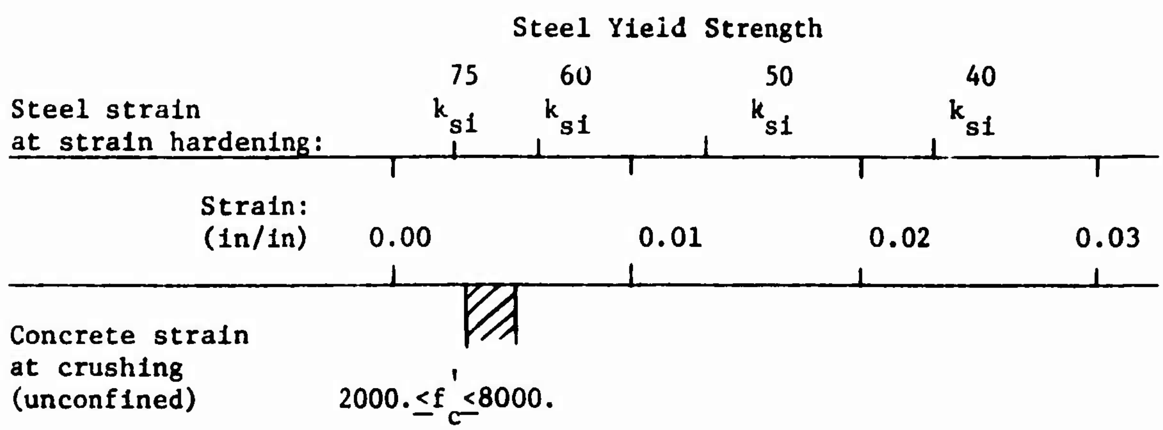
b. The process is most likely initiated by the expansion of the confined concrete near the ultimate stress level, effectively pushing the bars outward from their normal positions;

c. The bars are in a yielded state in compression, usually in the strain hardening range, at the time of buckling, [B.31];

d. Normal web reinforcement provides a restraint to displacement of the longitudinal bars at their contact points, thus providing a significant influence on the bending strength of a bar segment, [B.5,31].

The concrete expansion effect can be measured by its volumetric strain under compressive stress. The lateral strain and volumetric strain of unconfined uniaxially loaded concrete begin altered behavior at approximately 80% of ultimate strength to the extent that near the ultimate stress the volumetric strain has changed signs from compression to tension, (see Fig. B.8). This indicates that there is an expansion effect developed to push the bars out of line, [B.28].

An important effect is the combination of yielded steel and crushed concrete cover which allows the failure process to occur. To see the possibility of this combined effect, the steel strains at strain hardening are compared to the range of strain at which unconfined concrete crushing occurs:



If some allowance is made for the fact that the normal strain at a section is larger at the surface than at the bar location, it is reasonable to

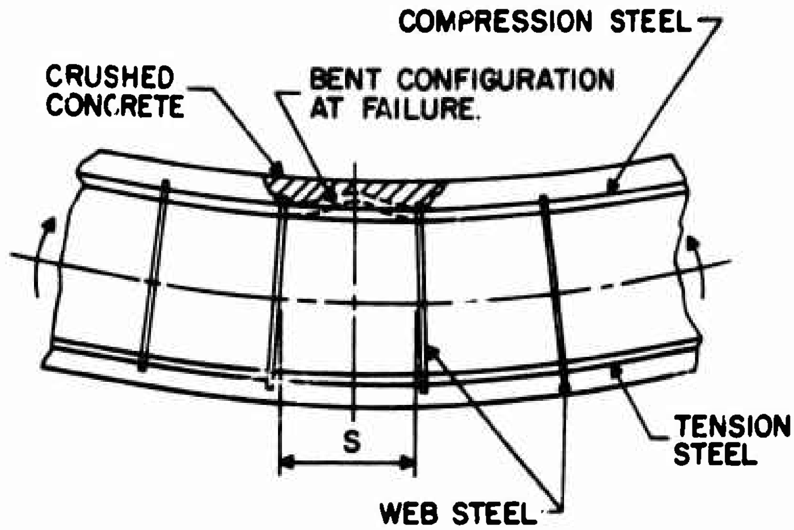


FIG. B.9: COMPRESSION ZONE FAILURE

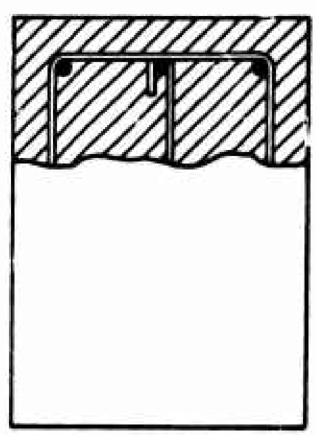


FIG. B. 10: DIRECT TENSILE RESTRAINT FOR LONGITUDINAL BARS.

state that when the steel enters the strain hardening range, the concrete cover has reached the crushing state.

In reference B.5, experiments were performed to study the behavior of the longitudinal steel as affected by compressive stress and lateral reinforcement. One conclusion reached was that the size of the lateral steel bar was not important for providing restraint against the outward displacement of the longitudinal bars; a positive direct tensile connection as shown in Fig. B.10 for a corner bar and an interior bar provide adequate restraint with the smallest diameter lateral bars.

In observations of this failure form in tests, it is difficult to determine whether the concrete crushing or the steel buckling initiates the final destruction [B.31].

The element model can predict the flexural behavior of an element into the nonlinear material range. However, it cannot predict the behavior after the buckling of the compression bars at a critical section since the compression zone has been destroyed in the region of buckling. Not only has a partial discontinuity been created at the section but the material uniformity along the length has been altered. The limit state is defined to be the buckling of compression bars stressed into the strain hardening range at the critical section of an element. The buckling condition for the outer layer of bars is considered sufficient for defining the limit state if there are multiple layers involved since an unknown stress concentration and redistribution is created due to this localized effect. For the member with web reinforcement, the crushing of the outer concrete is not considered to alter the behavior significantly. The element model is assumed to be valid up to the point of bar buckling.

The expression for predicting the critical stress for a uniform bar in simple compression is used to measure the buckling condition. To include the effect of the strain hardening state of the material, the tangent modulus property is included. The critical stress is defined by the following expression:

$$f_{cr} = C_2 \cdot \pi^2 \cdot \frac{E_t}{(S/K)^2} \tag{B.6}$$

where

f_{cr} = critical compressive stress in longitudinal reinforcing bar

E_t = tangent modulus for steel at f_{cr}

S = spacing of web reinforcement

K = radius of gyration of the bar

C_2 = end restraint coefficient.

C_2 = 1: pinned end condition

C_2 = 4: fixed end condition

This equation for f_{cr} was used in reference B.5 as a basis for determining the spacing of web reinforcement in compression members; a value of C_2 = 2 was used.

To implement this prediction, the model can compute the stress in a bar directly from a given strain state, and this value can be compared to a specific critical stress value. One bar size and one material are representative of the entire layer checked.

(1) Basic failure criterion: the relationship is based on the equation for critical stress of an initially straight uniform bar in simple compression:

$$f_{cr} - f_s \leq 0 \tag{B.7}$$

where

$f_{cr} = C_2 \pi^2 \cdot \frac{E_t}{(S/K)^2}$ defined in equation B.6;

f_s = compressive stress in longitudinal reinforcing bar.

If f_{cr} is expressed in terms of the bar diameter D, i.e. let $k = D/4$, and if the criterion is put in a dimensionless form, the expression can be re-written as:

$$C_2 \cdot \frac{\pi^2}{16} \cdot \left(\frac{D}{S}\right)^2 \cdot \frac{E_t}{f_s} - 1 \leq 0 \tag{B.8}$$

(2) Axial force effect: included in the strain state.

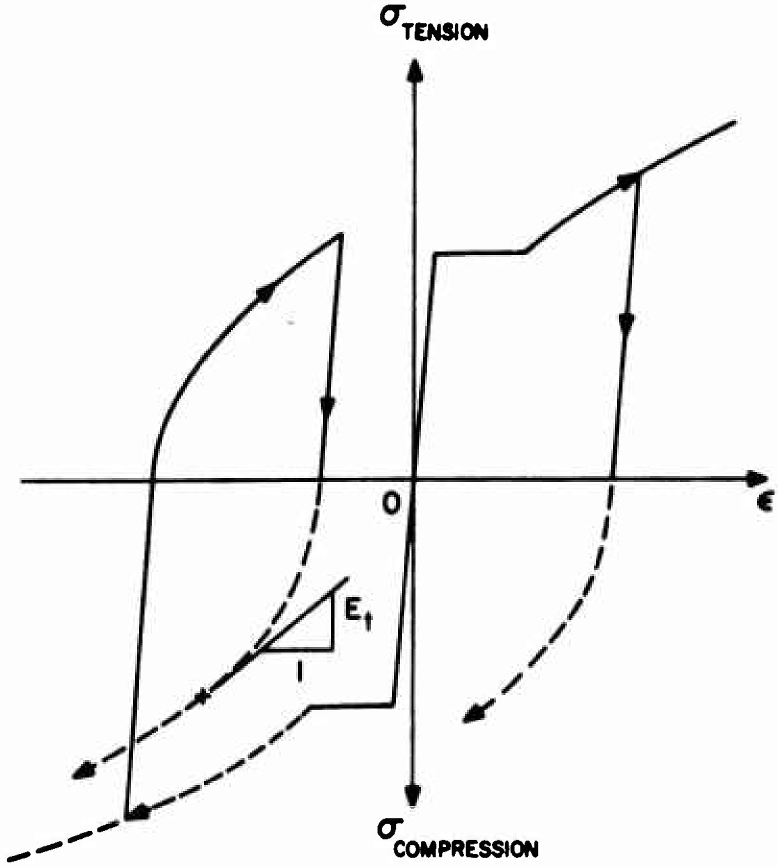
(3) Loading history effect: since the stress-strain response is uniquely defined for unloading and reloading of steel bars, the check can be made at any point where a tangent modulus exists. The check is applied for a compressive stress state in the strain hardening range for any cycle of loading. This is demonstrated in Fig. B.11.

(4) Modification:

a. Modification by coefficient is made through the constant C_2. This reflects the effects of various end restraints on the critical buckling stress. The end restraint on the segment between web bars is a function of the continuity of the bar and its freedom to deform. The modified equation is:

$$C_2 \cdot \frac{\pi^2}{16} \cdot \left(\frac{D}{S}\right)^2 \cdot \frac{E_t}{f_s} - 1 \leq 0 \tag{B.9}$$

$(1. \leq C_2 \leq 4.)$



----------- FAILURE CHECK MADE

FIG. B. 11: FAILURE CHECK FOR BAR BUCKLING.

$C_2 = 2$ is used as the lower bound value.

b. The user may choose to completely suppress this particular failure mode. By overriding this condition the member will maintain the original configuration of the steel for all stress-strain values.

5. Assumptions:

a. The steel bar is initially straight and uniformly compressed;

b. The steel bar cannot buckle until it is stressed into the strain hardening range of behavior;

c. All normal web reinforcement bars provide sufficient restraint to prevent lateral displacement of the longitudinal bars at the point of contact;

d. Buckling of the longitudinal steel bars initiates the destruction of the compression zone at the critical section.

B.3.1.4 STEEL FRACTURE

This failure mode is the fracture of the longitudinal steel caused by excessive tensile strain. Since this fracture is unafiected by web reinforcement in a member, the failure criterion is identical to the criterion developed in section B.3.1.2.

B.3.2 SHEAR-FLEXURE FAILURE

This category of failure defines the failure modes and criteria for the effects of shear stress and normal stress acting in combination within a member. The failure modes which require a failure criteria are shown in Fig. B.3.

The distinct form of behavior for this type of failure is a diagonal crack which more or less follows a path normal to the principal tensile

stress trajectories in the member. Although several similar cracks may be initiated, one crack becomes dominant as the load is increased. If web reinforcement exists at the crack location, sudden material separation is prevented.

The principal diagonal crack alters the member behavior by introducing a discontinuity. The crack is deep enough to disrupt the flexural characteristics within the element by reducing the size of the compression zone in a localized region. As a result, a redistribution of stresses is required to maintain equilibrium in the altered state, [B.10,21,35].

The discontinuous behavior of diagonal tension cracking in reinforced concrete members is associated with the weakness of concrete to principal tensile stress. The combination of normal tensile stress due to flexure, and shear stress due to variation in flexure can produce a more severe principal tensile state at a point than for conditions due to flexure alone. The tensile strength of concrete depends on local properties near the point of fracture in addition to the strain distribution, hence it is random in nature. This manifests itself in the unpredictability of specific locations and shapes of diagonal cracks.

This is in contrast to the more predictable behavior in bending of the same type of member. In pure bending, the member strength depends on the compression zone properties of concrete and the tensile strength of the longitudinal steel. The resulting crack formation in the tensile zone is of only secondary significance because these cracks are stabilized by a uniform compression zone.

B.3.2.1 DETECTION OF PRINCIPAL DIAGONAL CRACK LEADING TO FAILURE

If a diagonal crack occurs in a member without web reinforcement, there are three forms of behavior which describe the ultimate failure condition:

a. The crack may propagate through the entire section;

b. The crack may stop near a three dimensional compression zone, e.g., near a concentrated force or a support point; the final failure form is a destruction of the reduced compression zone above the crack;

c. The crack may propagate parallel to the tensile steel bars effectively removing their contribution from the element load resistance.

The ultimate failure conditions described by the crushing of the compression zone above the crack (b) and splitting along the longitudinal steel (c) are considered post-cracking behavior since they occur after the formation of the principal crack.

The significant factors which affect the shear cracking behavior have been determined principally from test results. Those factors considered most significant are:

a. the longitudinal steel percentage,

b. the tensile strength of concrete,

c. the dimensions of the cross section in comparison to the length of the member,

d. an axial force effect in addition to effects of lateral forces applied,

e. the location of the region of maximum shear force with bending,

f. the anchorage and bond characteristics of the longitudinal steel, and

g. the way loads are applied to a member, directly or indirectly.

These factors can be divided into three main groups: the initial shape and reinforcement of the member (a., c.), material properties (b., f.), and the internal stress distribution (d., e., g.). The steel properties are not significant because the steel does not yield before the formation of the significant crack unless the flexural effects are dominant [B.10]. The anchorage and bond characteristics of the longitudinal steel have the most influence on the post-cracking behavior.

The limitations imposed on a continuum model are based on physical conditions which significantly alter the dominant normal stress behavior predicted by the model. The discontinuous effects of a diagonal crack and the resulting stress concentration at the tip of the crack are not included. Therefore, the limit state is considered to be the initial formation of the principal diagonal crack. This precludes the consideration of the post-cracking behavior as limit states.

The element model measures the normal stress distribution along the length of the element. Flexural and axial stress effects are both derived from the given strain state. To get a measure of the shear stress effects, only indirect procedures are available: i.e., the shear force can be computed to satisfy equilibrium for the normal stress resultants, and a nominal measure of the stresses can be based on an average distribution equivalent to a shear force at a cross section. Consequently, the conditions causing the formation of a diagonal crack cannot depend upon stresses at a point (micro criterion); the true stress distribution is unknown. The crack prediction must be based on the nominal conditions

indirectly related to the stresses predicted by the model.

The indirect measure usually appears as an empirical relationship between gross element properties and nominal stress values. From an understanding of the uncertainties associated with the formation of the principal diagonal crack, it is understandable that the relationships developed for their prediction are inaccurate for generalized conditions. The criterion developed in this section incorporates the variables observed to be significant in tests, while accounting for the uncertainties by providing lower bound functions.

There are two important considerations in defining the failure criterion for diagonal tension cracking: (1) the equation for predicting the cracking strength of a member at a particular section for generalized conditions; and (2) the choice of a critical section at which to apply the equation. The prediction equation is discussed below; the application to a critical section is discussed in the implementation section (B.4).

The form for the failure criterion is

$$\frac{V_c(x)}{bd\sqrt{f'_c}} = \frac{v_c(x)}{\sqrt{f'_c}} = A + B\,\frac{pd}{\sqrt{f'_c}}\left|\frac{V(x)}{M(x)}\right| \tag{B.10}$$

v_c = nominal measure of cracking stress at station x

V_c = cracking force at a section x

b = gross cross section width

A,B = constants to be defined by data

f'_c = ultimate cylinder strength of concrete

($\sqrt{f'_c}$ is a measure of tensile strength)

p = longtudinal steel percentage = A_s/bd

$V(x), M(x)$ = shear force and bending moment at a section x where v_c is computed

d = effective depth of longitudinal steel.

The form of this equation is identical to that presented in reference B.30, and it has also been used in modified versions by others [B.10, 17]. (Note: other equations have been developed for computing v_c, e.g., references B.21,37, but the fit of data is no better in general.) This equation defines the nominal shear stress at the initial crack formation for a specific location x as a function of the element dimensions, reinforcement, and material properties (p, b, d, f'_c), and as a function of the internal stress distribution ($V(x)/M(x)$) evaluated at the same location x. The ratio (V/M) measures the effect of the shear force and the bending moment on the principal tensile stress, and hence on the crack formation. As a ratio, this influence is translated into a distance measurement, from the point where $M = 0$ to the crack location.

In order to evaluate the constants A and B consistent with these characteristics, the $V(x)$ and $M(x)$ values must be known at the crack location, in addition to the other factors. The two basic parameters,

$$F_1(x) = \frac{V_c(x)}{bd\sqrt{f'_c}} \quad \text{and} \quad F_2(x) = 1000 \cdot \frac{pd}{\sqrt{f'_c}} \cdot \left| \frac{V(x)}{M(x)} \right|$$

can be evaluated for specific test cases for a wide variation in each factor. The choice of a specific function to fit the data will establish A and B.

The process chosen here is to develop a basic equation using only the results from simple beam tests with concentrated loads since they represent,

in some form, all concentrated load conditions. In addition, there are many test results of this form available. This basic equation can then be checked against other conditions for more general applicability. The basic data is from reference B.30 and is plotted in Fig. B.12. The lower bound function is shown on the figure and can be expressed as:

$$F_1(x) = 1.5 + 3.5 \ F_2(x) \leqq 3.0 \qquad \text{(B.11)}$$

where A = 1.5 and B = 3.5 define the intercept distance and the slope of the function.

The method of load application is significant to the cracking strength particularly for small (a/d) values, where a is the shear span length, [B.12]. Such refinement is not considered appropriate in this behavior prediction for lack of quantitative measurements and precise characterization of the physical loads actually applied. It should be noted, however, that for most beam dimensions and loading ($F_2(x)<0.3$), the (a/d) values are larger, and the support and load effects are not significant in the region of the crack.

The check of the lower bound function for the cases of simple beams with uniformly distributed loads and for restrained and continuous beams with concentrated loads are shown separately in Figs. B.13 and B.14, with reasonable acceptance.

(1) Basic failure criterion: defined by the lower bound function developed:

$$F_1(x) - F(x) \leqq 0 \qquad \text{(B.12)}$$

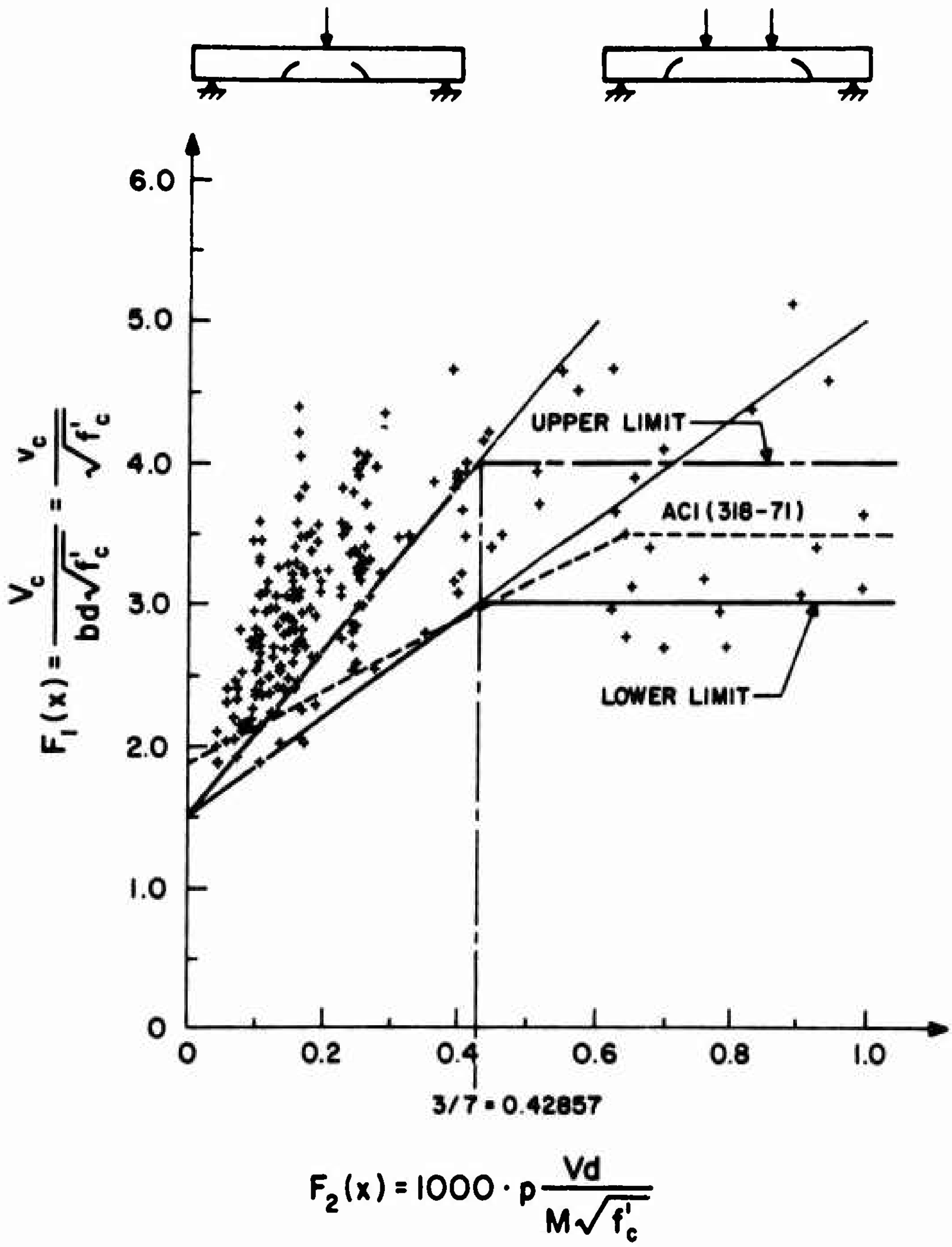


FIG. B. 12: DATA FOR INITIAL SHEAR CRACKING - (A) (PLOTTED DATA FROM REF. [B. 30] TABLES 5-1, 2, 8, 9, 12.)

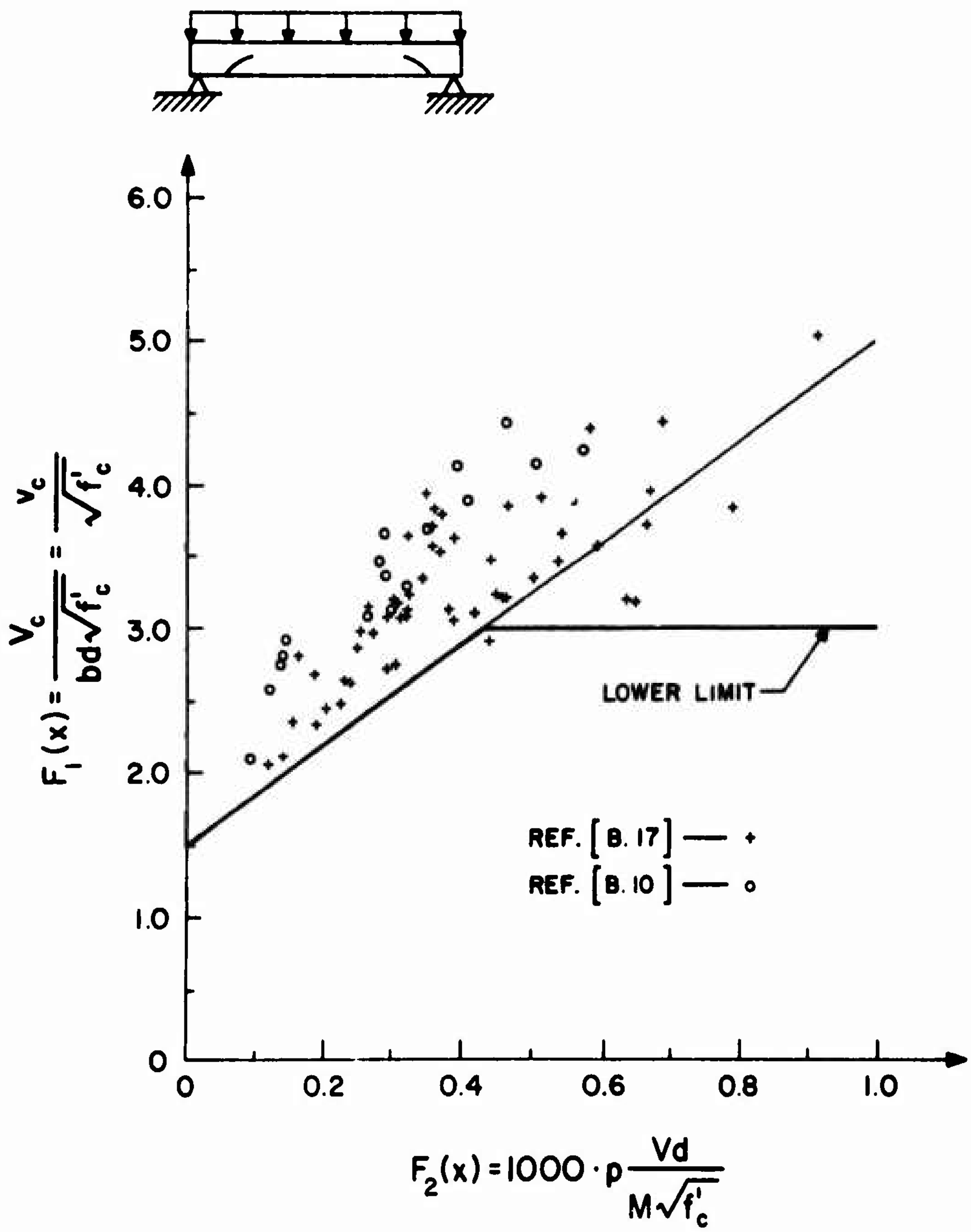


FIG. B. 13: DATA FOR INITIAL SHEAR CRACKING-(B.)
(PLOTTED DATA FROM REF. [B. 10, 17]).

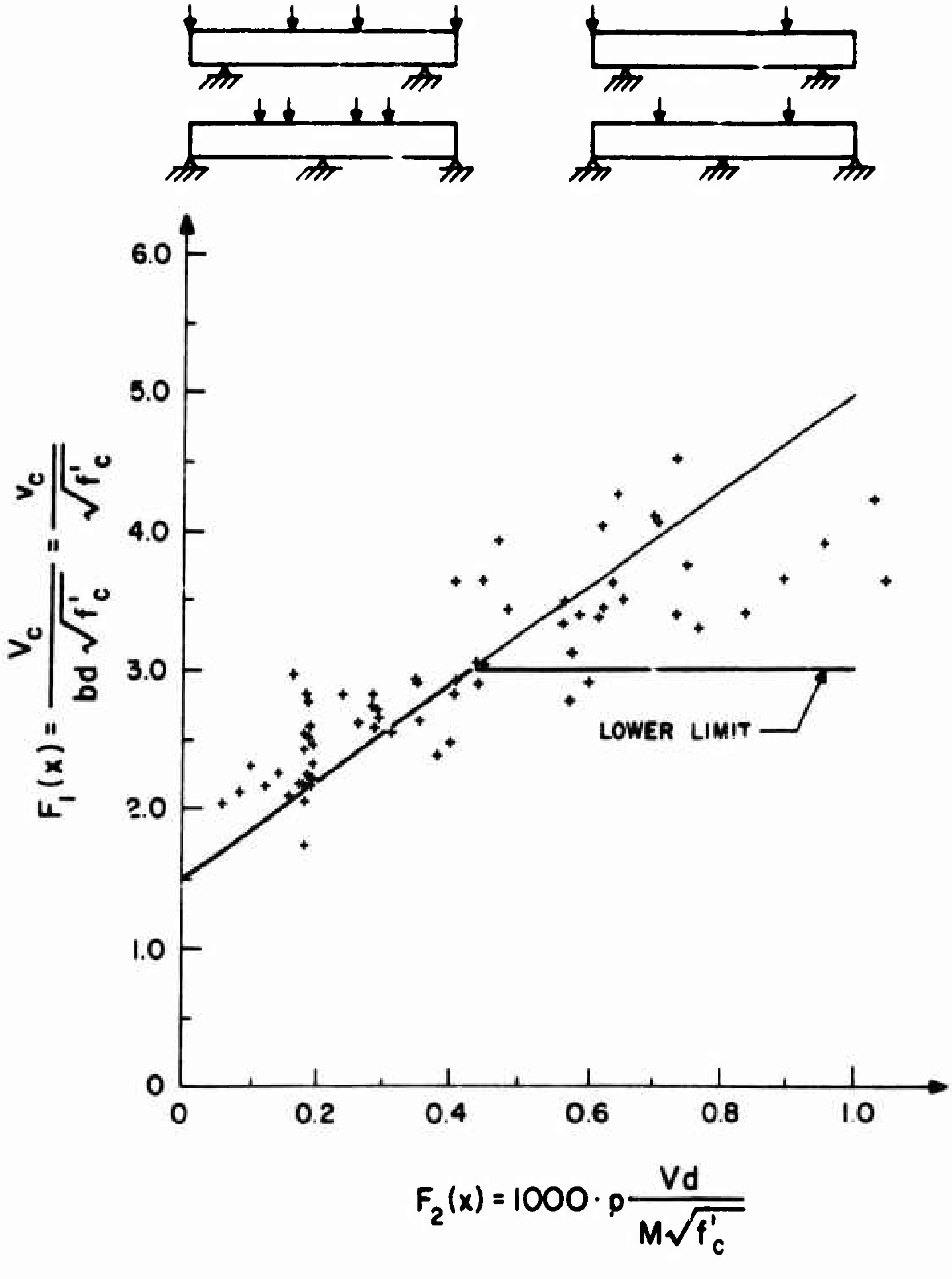


FIG. B. 14: DATA FOR INITIAL SHEAR CRACKING - (C) (PLOTTED DATA FROM REF. [B.30] TABLES 5-3, 4.)

where

$$F_1(x) = 1.5 + 3.5 \cdot F_2(x) \leq 3.0$$

$$F(x) = V/bd\sqrt{f'_c}\ .$$

F(x) is a nominal measure of the actual shear force at the critical section.

The equation is considered applicable to any concentrated load condition for straight elements because the shear and moment functions can be broken down into equivalent simple beam segments when the sections of zero moment are located. Each segment of constant shear can be examined separately.

(2) Axial force effect: this effect is to measure the change in the basic criterion caused by the application of a tensile or compressive axial force. The effect is reflected in the alteration of the principal stress trajectories in the element. A compressive force causes a larger compression zone which forces the diagonal crack to bend over at a shallower depth from the tension face. A tensile force causes the compression zone to be smaller in depth resulting in a deeper and less inclined crack [B.10, 14].

The axial force influence is not symmetric, particularly for larger values. In the limiting case, where the axial force dominates the behavior, the effects of compression and tension are distinctly different. (This behavior is discussed in section B.3.3.).

The choice for an equation to predict the axial force affect was made on the basis of the following points:

a. It is easier and more efficient to work with strains and stresses from the model rather than stress resultants.

b. The equation should be numerically easy to apply and easy to modify by coefficients.

c. Prediction of the axial force effects should be no less accurate in general application than the currently accepted design equations.

The two alternative approaches to specifying a prediction equation were: (1) to use the current ACI Code equation (ACI Standard 318-71),

$$V_c = 1.9\sqrt{f'_c} + 2500\ p \cdot \frac{Vd}{M - N \cdot \left(\frac{4h-d}{8}\right)} \tag{B.13}$$

or (2) to develop a different equation and check it with code equation.

The choice was made to develop a different equation for the following reasons:

a. The ACI Code equation required the computation of stress resultants with no alternative;

b. To modify the equation with a coefficient for the effect of N, and to determine lower and upper limits to its variation is difficult in the form presented by the Code equation.

c. A different equation could be developed to more closely fit the desired qualities;

d. There appeared to be sufficient published test results as a basis for the equation development and for testing its accuracy for both tension and compression effects.

The initial form of the equation was based on a form suggested in reference B.10 , i.e.,

$$\frac{V_{cn}(x)}{bd\sqrt{f'_c}} = [1.5 + 3.5\ F_2(x)] \cdot [1 - \alpha \cdot \frac{N(x)}{V_{cn}(x)}] \tag{B.14}$$

The new terms are defined to be:

$V_{cn}(x)$ = cracking shear force at section x with axial force

$N(x)$ = axial force (tension position)

α = coefficient for axial force effect.

This equation was modified because N/V_{cn} did not correctly measure a change in the cracking strength for all cases. This is due to the fact that V_{cn} is influenced by N; i.e., V_{cn} increases for N compression, and decreases for N in tension. This equation is more reasonable if there is a fixed ratio for N/V_{cn} up to the cracking strength.

The modified form uses $V_c(x)$ as the reference force for N(x), where $V_c(x)$ is the cracking force without an axial force effect; i.e.,

$$\frac{V_{cn}(x)}{bd\sqrt{f'_c}} = [1.5 + 3.5\ F_2(x)] \cdot [1. - \alpha \frac{N(x)}{V_c(x)}] \qquad (B.15)$$

Noting that:

$$F_1(x) = \frac{V_c(x)}{bd\sqrt{f'_c}} = 1.5 + 3.5\ F_2(x) \qquad \text{(equation B.11)}$$

and nondimensionalizing N and V_c by the factor $bd\sqrt{f'_c}$, equation (B.15) can be written:

$$\frac{V_{cn}(x)}{bd\sqrt{f'_c}} = \frac{V_c(x)}{bd\sqrt{f'_c}} \cdot [1. - \alpha \cdot \frac{N(x)/bd\sqrt{f'_c}}{V_c(x)/bd\sqrt{f'_c}}]$$

and finally,

$$\frac{V_{cn}(x)}{bd\sqrt{f'_c}} = \frac{V_c(x)}{bd\sqrt{f'_c}} - \alpha \cdot \frac{N(x)}{bd\sqrt{f'_c}} \qquad (B.16)$$

For $F_3(x) = \dfrac{V_{cn}(x)}{bd\sqrt{f'_c}}$ and $F_4(x) = \dfrac{N(x)}{bd\sqrt{f'_c}}$

equation (B.16) can be written:

$$F_3(x) = F_1(x) - \alpha\, F_4(x) \tag{B.17}$$

The limitation ($\leq$ 3.0) shown in equation B.11 is omitted in the derivation, but it is to be incorporated in the application.

To evaluate the lower limit for the coefficient α, the equation B.17 was rearranged as shown below:

$$\frac{v_{cn} - v_c}{v_c} = \alpha \left| \frac{N}{V_c} \right| \tag{B.18}$$

where $v_{cn} = \dfrac{V_{cn}}{bd}$ and $v_c = \dfrac{V_c}{bd}$. These two basic factors are evaluated for a series of tests and plotted in Fig. B.15. The lower limit function is shown as $\alpha = 0.10$ for both tension and compression. A reasonable average value was chosen for the lower limit of α to avoid compounding lower bound features into one equation. The basic equation for V_c was previously defined to be a lower bound fit of data.

The final justification for the implementation of the derived equation is based on its ability to predict the observed trends in test results. The derived equation was applied to the data in reference B.14 which included tests for tension, compression and zero axial force. The results are shown in Fig. B.20 as a comparison with test values; on the same plot, the ACI Code equation is applied to the same set of data. It is clear that the derived equation is as valid as the Code equation for measuring all three axial force effects. The data from reference B.14 is shown to be incorporated into the derivation of the coefficient α and

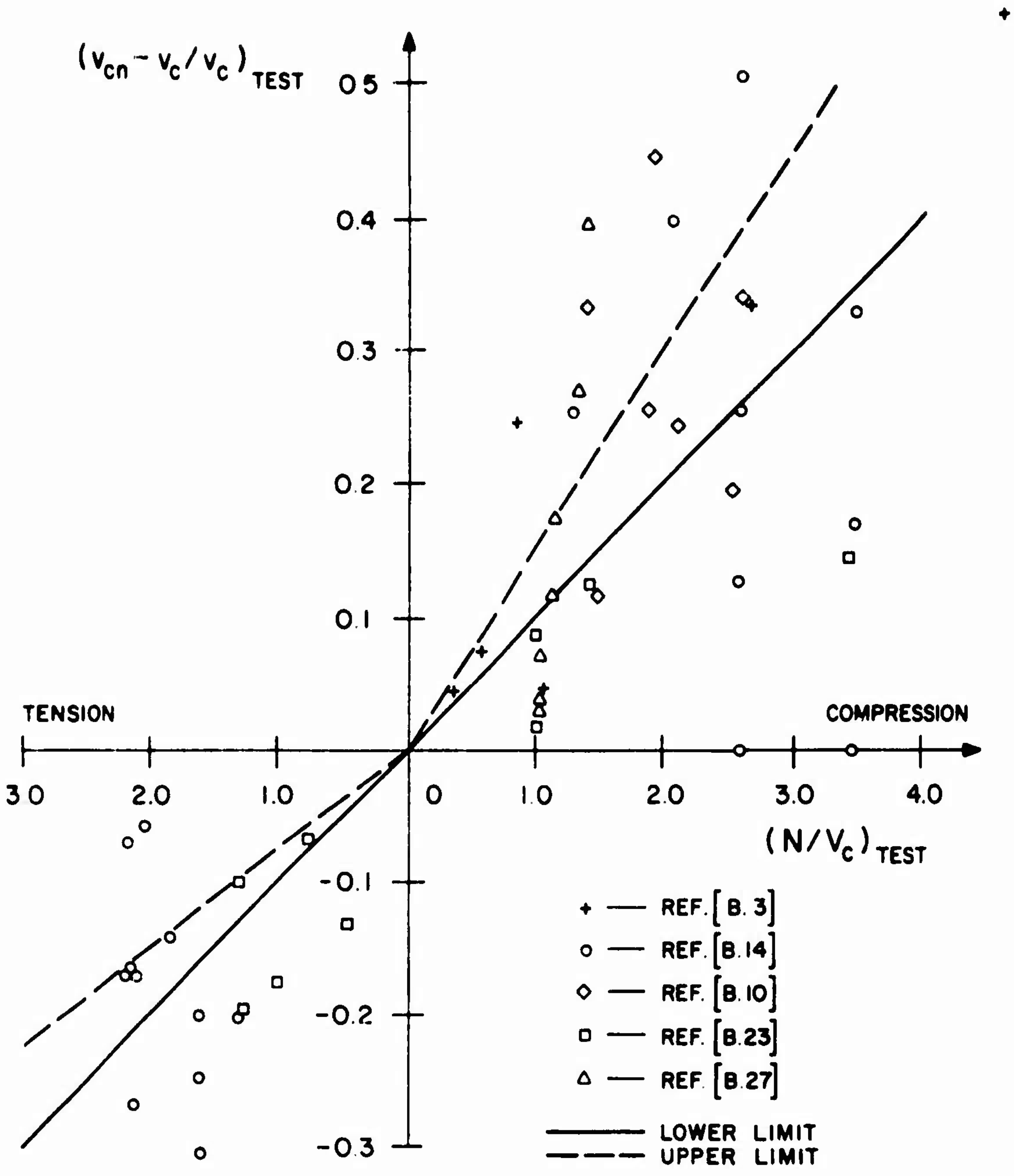


FIG. B.15: DATA FOR INITIAL SHEAR CRACKING WITH AXIAL FORCE EFFECTS. (DATA FROM REF. [B.3, 14, 10, 23, 27]).

in checking the equation. However, the data points on Fig. B.15 corresponding to reference B.14 were plotted after the equation check was made so that an independent set of tests were available to make the check.

In reference B.14 , a recommendation is made to incorporate a cut-off value for large tensile forces; i.e. for N (tension) > $4\sqrt{f'_c}$, V_c is to be taken equal to zero. It is argued that too much preliminary cracking caused by the tensile force destroys the shear resistance of the member. There is no such cut-off recommended with the failure criteria for the following reasons:

a. All of the test results from reference B.14 do not support the recommendation made;

b. The recommendation is based on one test for which the axial load was applied first, then the lateral load up to cracking, which is not realistic in an actual structure;

c. The plotted results of the derived equation in Figure. B.20 did not indicate any trend to support the cut-off limit.

The basic failure criteria with axial force effect is:

$$F_3(x) - F(x) \leq 0 \qquad \text{(B.19)}$$

where $F_3(x) = F_1(x) - \alpha \cdot F_4(x)$

$F(x) = V(x)/bd\sqrt{f'_c}$.

(3) Loading history effects: the effect of reversal of load on the shear strength of members was tested and results presented in reference B.2 . Two conclusions are significant:

a. It was found that cracking load associated with a reverse load is of the same order of magnitude as the initial cracking load, even in the presence of the initial diagonal crack;

b. The repetition of a few high reverse loadings does not cause a significant decrease in the strength of beams failing in shear as compared to monotonic loading failure.

Therefore, it is assumed that the developed criteria is valid for any loading history consistent with a few cycles of loading effects.

(4) Modification:

(a) Modification by coefficient: the suggested modification is made by increasing the slope of the basic function, equation B.11, but not the point of intersection at $F_1(x) = 1.5$. The horizontal cut-off line can be defined by the intersection with $F_2(x)=3/7$ for each slope. The form for modification is defined below:

$$F_3^*(x) = \left([1.5 + C_3 \cdot F_2(x)] \leq \frac{3(7+2C_3)}{14} \right) - \alpha \cdot F_4(x) \qquad \text{(B.20)}$$

and α (tension) $= \alpha_t = 0.025(4.-C_4)$

α (compression) $= \alpha_c = 0.050(2.+C_4)$

The range for coefficients C_3 and C_4 are defined by:

$(3.5 \leq C_3 \leq \frac{35.}{6})$

$(0. \leq C_4 \leq 1.)$

The upper limit function for $N(x) = 0$ is shown on Fig. B.12 corresponding to $C_3 = 35./6.$; the upper limit for α_t and α_c are shown on Fig. B.15 for $C_4 = 1.0$.

(b) This failure criterion can be overridden since it is an indirect measure of a possible discontinuity. If it is ignored, the user

should realize that only the limitations of the flexural behavior are in effect, and unrealistic behavior is possible.

(5) Assumptions:

The shear crack in a member with several shear spans (see Fig. B.18), can be predicted by a criteria based on simple beam tests with or without an axial force.

B.3.2.2 DETECTION OF PRINCIPAL DIAGONAL CRACK PLUS YIELDING OF WEB REINFORCEMENT LEADING TO FAILURE.

If a diagonal crack occurs in a member with web reinforcement, material separation is prevented by the bars intersected by the crack. The only form of web reinforcement considered is the closed form normal to the longitudinal reinforcement.

There are two behavior conditions which describe ultimate failure states for a member with web reinforcement [B.14, 30]:

a. The diagonal crack may propagate through the member and cause sudden yielding of the web reinforcement at the crack location;

b. The diagonal crack may be contained by the web reinforcement until crushing of the reduced compression zone at an increased load before or after the web steel yields.

In addition, experimental tests have shown that the stress in the web bars at the crack remain very small until the significant diagonal crack forms [B.7, 9, 11, 25]. This is due to the fact that brittle materials possess low extensibility [B.36].

To predict shear-flexure failure of a member with web reinforcement requires some measure of the behavior after the formation of the diagonal crack. This means that the stress in the web bars intersected by the crack must be predicted since subsequent behavior depends on the containment of the crack by these bars. However, to predict the destruction of the compression zone above the crack requires some knowledge of the size of the compression zone and the stress concentration effects in the localized region. This is beyond the model capability. Therefore, the limit state considered for this failure mode is the detection of the yielding of web reinforcement in the range of the crack. As long as the web bars are below

the yield stress, the element is considered to be reliable. Beyond the yielded state, the element model is considered to be invalid, and the behavior cannot be accurately predicted.

An approximate measure of the web steel stress can be computed indirectly. The form of a prediction expression can be developed on the basis of measured stirrup strain during loading (See Fig. B.16). Accordingly, the assumed stress variation is zero up to the cracking load, and thereafter is a linear function of the applied load;

a_1, a_2 = constants

P_c = cracking load

ε_v = web steel strain

P = applied load

$$\varepsilon_v = a_1 + a_2 P$$

$$a_1 + a_2 P_c = 0$$

$$\varepsilon_v = a_2 (P - P_c) \qquad \text{(B.21)}$$

If it is also assumed that the shear force at a section is proportional to the applied loads, then:

C = constant

V = shear at section

V_c = cracking shear at section

f_v = web steel stress

E = linear modulus for web steel

$C' = C \cdot E$ = constant

$$\varepsilon_v = C (V - V_c)$$

$$\varepsilon_v = \frac{f_v}{E}$$

$$f_v = C' (V - V_c) \qquad \text{(B.22)}$$

This corresponds to the same form accepted by the ACI code (318-71) and used by others in describing test results [B.14, 17, 30, 38]; i.e.:

b = width of member

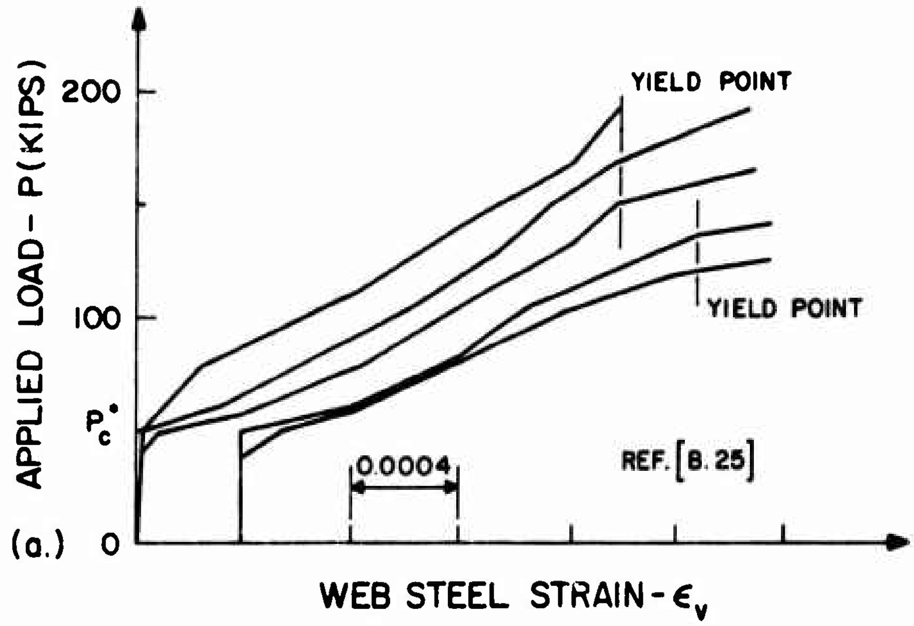
d = effective depth

A_v = total web steel area at one section = A_b x no. of legs

$$v = rf_{vy} + v_c$$

$$r = \frac{Av}{bs} ; \; v = \frac{V}{bd} ; \; v_c = \frac{Vc}{bd}$$

$$f_{vy} = \frac{S}{dA_v} (V - V_c) \qquad \text{(B.23)}$$



$^*P_c$ = LOAD AT CRACKING

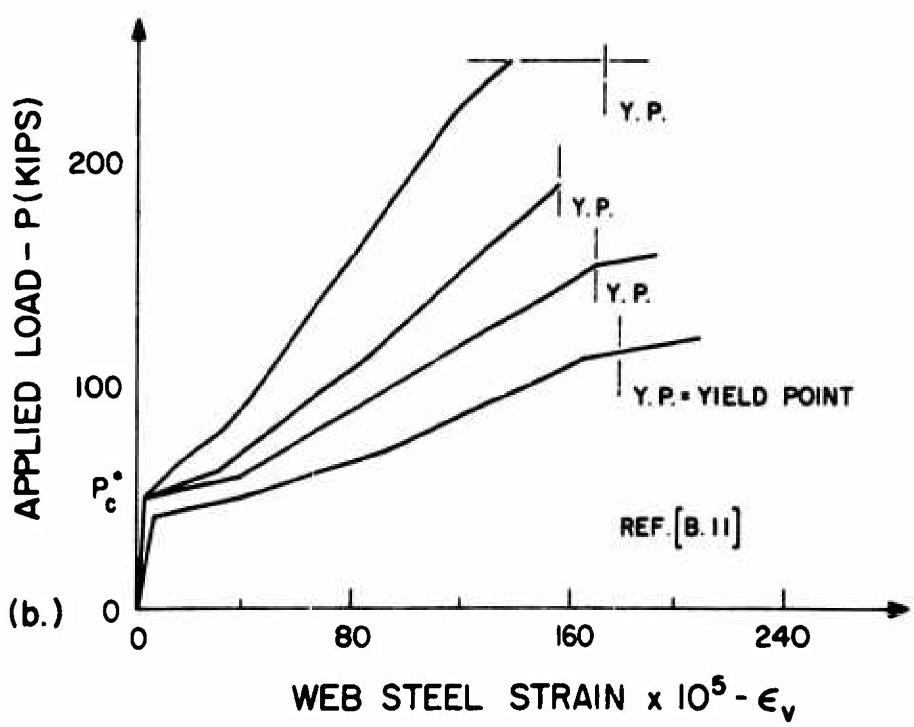


FIG. B.16: WEB STEEL STRAIN AS A FUNCTION OF APPLIED LOAD (DATA FROM REF. [B. 11, 25])

S = web bar spacing

f_v, V, V_c = defined above

If it is assumed that the crack is inclined to the longitudinal axis at approximately 45°, then (d/S) is a measure of the number of bars (n) crossing the crack; i.e., n = d/S.

According to observations, all of the web bars crossing a physical crack are affected by that crack. In addition, by the time the ultimate behavior state is reached, all of these bars can yield, [B.14,7]. All of the bars at a crack should be included in the total effect, although each stress cannot be predicted independently. Therefore, the total area, $(n.A_v)$, is used in the equation.

The average stress value for the web bars crossing a crack for any value of shear force is given by the equation for f_v (equation B.22) with $C' = S/dA_v$; i.e.:

$$f_v = \frac{S}{dA_v} \cdot (V-V_c)$$

To introduce a lower bound measurement for yielding, the data plotted in Fig. B.17 is used. The lower limit function can be defined by the equation:

$$f_{vy} = \frac{4}{3} \cdot \frac{S}{dA_v} \cdot (V-V_c) \tag{B.24}$$

which is valid at yielding conditions only. To introduce f_v at any stress state, the relationship must be made an inequality; i.e.:

$$f_{vy} > \frac{4}{3} \cdot f_v \text{ : before yielding}$$

$$f_{vy} \leqq \frac{4}{3} \cdot f_v \text{ : at or after yielding.}$$

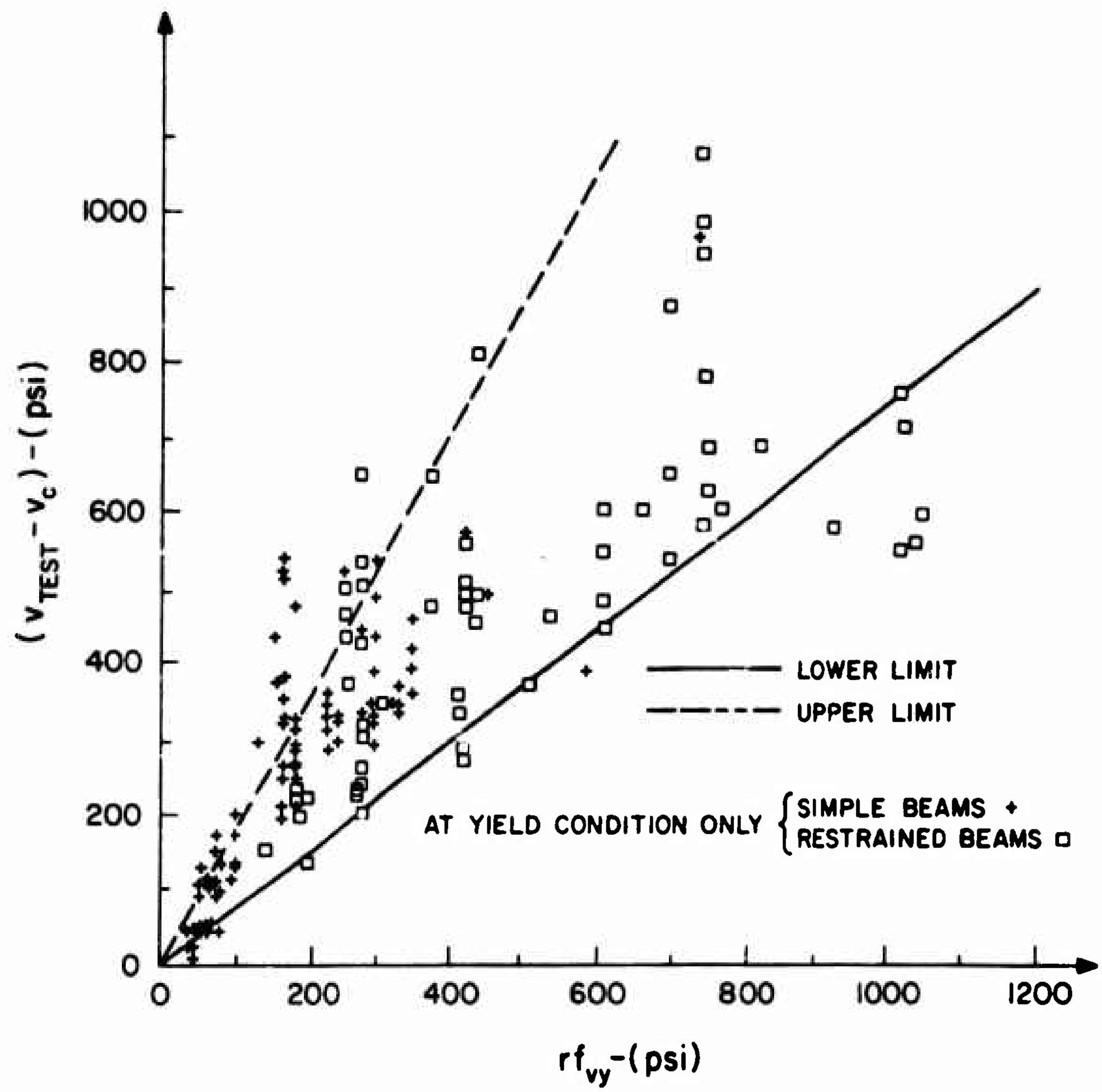


FIG. B. 17: ULTIMATE STRENGTH MEASUREMENT WITH NORMAL WEB REINFORCEMENT (DATA FROM REF. [B. 30]).

(1) Basic failure criterion : based on an equation to approximate the web steel stress at a crack:

$$f_{vy} - \frac{4}{3} f_v \leq 0 \qquad (B.25)$$

where f_{vy} = yield stress in web reinforcing bars

$$f_v = \frac{S}{dA_v} (V-V_c) .$$

By introducing $F(x)$ and $F_1(x)$ defined in section B.3.2.1, and arranging the expression in a dimensionless form, it can be re-written as shown below:

$$1. - \frac{4}{3} \cdot \frac{Sb}{A_v} \cdot \frac{\sqrt{f'_c}}{f_{vy}} \cdot [F(x) - F_1(x)] \leq 0 . \qquad (B.26)$$

An upper limit for the value of $(V-V_c)$, as recommended by the ACI Code, is not included in the criterion for two reasons:

(a) Test results indicate that the upper limit is not consistent with physical behavior [B.14];

(b) A natural upper limit to failure for large values of $(V-V_c)$ is available through the flexural failure criteria .

(2) Axial force effect: this is to measure the effect of an axial force, tension or compression, on the physical behavior of web reinforcement after cracking. Although data is scarce, reference B.14 provides results to indicate that the contribution of web reinforcement to the total strength is approximately independent of both the axial force and the ratio (a/d). These results were for limited variations of the parameters, but they form the basis for the criterion statement. The measure of the limit state for the shear strength of a member with web reinforcement as defined above is assumed to be valid for a moderate axial force, either compression or tension. It should be mentioned that some effect is already built into

the expression through V_c as defined by V_{cn} in section B.3.2.1. The quantity that is assumed to remain unaffected is $(V-V_c)$ or $(V-V_{cn})$. The expression which includes the axial force effect should be expressed as follows:

$$1 - \frac{4}{3} \cdot \frac{sb}{A_v} \cdot \frac{\sqrt{f'_c}}{f_{vy}} \cdot [F(x) - F_3(x)] \leq 0 \tag{B.27}$$

where $F_3(x) = V_{cn} / bd\sqrt{f'_c}$

(3) Loading history effects: the question is whether unloading or reversal of loading has any effect on the basic behavior of web reinforcement. The results presented in reference B.2 included behavior of beams with web reinforcement subjected to a few reversed loading cycles. The conclusion stated in section B.3.2.1, part 3 is applicable to this part also; i.e., the repetition of a few reversed loadings does not cause a significant decrease in the strength of beams failing in shear as compared to monotonic loading failure. The criterion stated remains valid with load reversal.

(4) Modification:

(a) Modification by coefficient: the web strength criterion can be altered by a coefficient which changes the slope of the function shown in Fig. B.17; i.e.,:

$$f_{vy} = C_5 \cdot \frac{s}{dA_v} \cdot (V-V_c)$$

The suggested relationship developed in reference B.14 to fit a specific set of data was defined to be:

$1.75rf_{vy} = v_u - v_c$, where v_u = ultimate shear stress.

This equation is plotted on Fig. B.17 and it appears to be a suitable upper bound. Accepting this function as defining an

upper bound value for C_5, the modified form of the criterion can be written as:

$$1. - C_5 \cdot \frac{sb}{A_v} \cdot \frac{\sqrt{f'_c}}{f_{vy}} \cdot [F(x) - F_3^*(x)] \leq 0 \qquad (B.28)$$

for $(4/7 \leq C_5 \leq 4/3)$

and $F_3(x)$ is included in its modified form $F_3^*(x)$, (equation B.20).

(b) this criterion can be overridden since it is an indirect measure of the web steel stress. If it is ignored, only the flexural behavior limitations are in effect.

(5) Assumptions:

(a) Web bars are anchored sufficiently to insure yield strength development;

(b) Element model remains valid after the formation of the diagonal crack up to the web yield state;

(c) The web steel strain is linearly related to the applied load after crack formation;

(d) All bars affected by the crack yield at the measured limit state.

B.3.3 AXIAL FORCE FAILURE

The limit states developed in this failure category relate to the discontinuities encountered in the behavior of a member dominated by normal stresses caused predominantly by an axial force. The distinction between this category and the category for normal stress effects due to flexure (B.3.1.) is that the point of zero strain at a specified section falls outside of the physical dimensions. Hence, the state of stress is either tension or compression over the entire cross section. The failure modes which require a failure criteria are shown in Fig. B.4.

The failure modes are similar to those for flexure; i.e., compressive failure means either crushing of the concrete or the simultaneous crushing and bar buckling in the critical compression zone; and tensile failure is the fracture of longitudinal bars. Therefore, the same failure criteria are applicable. In the axial force category, the secondary stresses are caused by bending. But since both are included in the normal strain state, these secondary effects are automatically accounted for in the failure criteria. This is similar to the secondary axial force effects automatically included in the flexural failure criteria.

B.3.3.1 CONCRETE CRUSHING

Failure criterion: same as section B.3.1.1.

B.3.3.2 STEEL FRACTURE

Failure criterion: same as section B.3.1.2.

B.3.3.3 BAR BUCKLING AND CONCRETE CRUSHING SIMULTANEOUSLY.

Failure criterion: same as section B.3.1.3.

B.3.3.4 STEEL FRACTURE

Failure criterion: same as section B.3.1.2.

B.3.4 SUMMARY OF FAILURE CRITERIA

Basic Form of Failure Criteria Expressions:

[(Specific Criterion Value) - (Computed Model Value)] $\leqq 0$

<u>All criteria are expressed in a dimensionless form except for B.3.1.1 as noted.</u>

<u>(B.3.1) Flexural Failure</u>

(B.3.1.1) Concrete Crushing: $\epsilon^*_{f1} - \epsilon_c \leqq 0$

ϵ_c - Compression

$$\varepsilon^*_{f1} = C_1 \cdot \left[\left(\frac{3 + 0.002f'_c}{f'_c - 1000} \right) \leqq 0.0035 \right]$$

$(1. \leqq C_1 \leqq 1.23)$

Default: $C_1 = 1$

override: not possible

Note: Constants in the expression for ε^*_{f1} are not dimensionless, even though the total expression is dimensionless.

(B.3.1.2) Steel Fracture: $\varepsilon_{f2} - \varepsilon_s \leqq 0$

ε_s - tension

ε_{f2} = maximum strain value defined for stress-strain function.

override: not possible

(B.3.1.3) Bar Buckling and Concrete Crushing Simultaneously:

$$C_2 \cdot \frac{\pi^2}{16} \cdot \left(\frac{D}{S}\right)^2 \cdot \left(\frac{E_t}{F_s}\right) - 1. \leqq 0$$

f_s - compression

$(1. \leqq C_2 \leqq 4.)$

Default: $C_2 = 2$

override: possible

(B.3.1.4) Steel Fracture: (Same as B.3.1.2)

(B.3.2) Shear-Flexure Failure

(B.3.2.1) Detection of the Principal Diagonal Crack Leading to Failure:

$$F_3^*(x) - F(x) \leq 0$$

$$F_3^*(x) = \left\{[1.5 + C_3 \cdot F_2(x)] \leq \frac{3(7 + 2C_3)}{14}\right\} - \alpha \cdot F_4(x)$$

$(3.5 \leq C_3 \leq 35./6.)$

Default: $C_3 = 3.5$

$$\alpha \begin{cases} \text{Tension:} & \alpha_t = 0.025(4. - C_4) \\ \text{Compression:} & \alpha_c = 0.050(2. + C_4) \end{cases}$$

$(0. \leq C_4 \leq 1.)$

Default: $C_4 = 0$

override: possible

(B.3.2.2) Detection of the Principal Diagonal Crack Plus Yielding of the Web Reinforcement Leading to Failure:

$$1. - C_5 \cdot \frac{sb}{A_v} \cdot \frac{\sqrt{f'_c}}{f_{vy}} \cdot [F(x) - F_3^*(x)] \leq 0$$

$(4/7 \leq C_5 \leq 4/3)$

Default: $C_5 = 4./3$

override: possible

(B.3.3) Axial Force Failure

(B.3.3.1) Concrete Crushing: (same as B.3.1.1)

(B.3.3.2)	Steel Fracture:	(same as B.3.1.2)
(B.3.3.3)	Bar Buckling and Concrete Crushing Simultaneously:	(same as B.3.1.3)
(B.3.3.4)	Steel Fracture:	(same as B.3.1.2)

B.4 IMPLEMENTATION

The application of the failure criteria is made at an equilibrium configuration for the system and at critical sections within each element. The implementation defines the critical sections used and describes the general application procedure.

B.4.1 ASSUMPTIONS

In addition to the assumptions made for the various failure criteria developed in B.3, the following assumptions are made with respect to the implementation of these criteria:

1. A single element has uniform reinforcement properties, in longitudinal and web steel, over the entire length. If a member has variable reinforcement properties, e.g., a region with web reinforcement and a region without, the failure criteria will be utilized more effectively by modeling the member with more than one element to reflect, at least approximately, the actual reinforcement uniformity.
2. General beam shear-flexure behavior can be effectively related to criteria based on simple beam test results by defining analogous segments within the model. The analogous segments, or shear spans, are defined by lengths measured from a point of zero moment to a nodal point, or from one nodal point to another. (Refer to Fig. B.18).
3. The most probable section for failure in an element is defined by the critical section. A critical section is prescribed for each failure criteria, and only the critical sections in each element

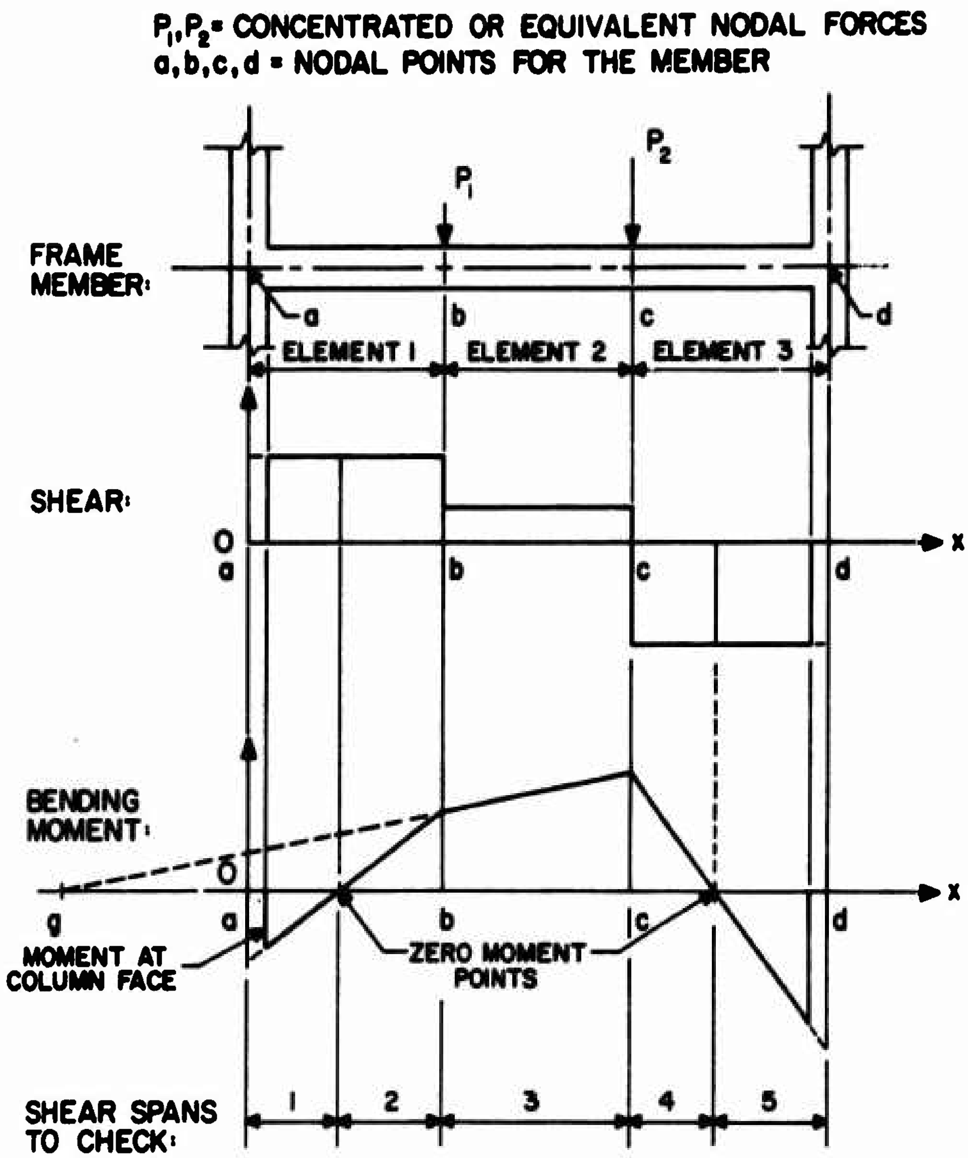


FIG. B. 18: POSSIBLE FRAME MEMBER STATE WITH SHEAR SPANS DEFINED.

are checked for failure.

4. An element defined with a length less than the effective depth d will not be checked for shear failure, but will be checked for flexural and axial failure.

5. The equivalent distance from zero moment to the critical section at the crack location, measured by $(V(x)/M(x))$ in the shear cracking equation, is the same regardless of the superimposed uniform strain caused principally by an axial force.

B.4.2 CRITICAL SECTIONS

Critical sections are the prescribed section locations where the various criteria are applied. It represents, in each case, the most probable location for failure consistent with the model representation of the actual member and applied loading. In applying the criteria, each element tested is treated as an independent unit, even though several elements may be joined together to represent a single physical member.

Failure due to flexural and axial force effects are related to maximum normal strain states in an element. For the element model used, this state occurs at one of the end sections. The end sections are then the critical sections for these two criteria, defined in B.3.1 and B.3.3. There is one exception to this definition: when an element has one or both nodal points located at a junction of two members normal to each other, e.g.,a horizontal member intersecting a column member, the critical section is defined at the face of the normal member rather than at the nodal point. This can be seen in Fig. B.18 at nodal point a and d.

Failure due to shear effects, with criteria defined in B.3.2, is detected by an indirect measure that includes material properties,

dimensions of the cross section, and a measure of shear span length. The critical section is defined by a conservative estimate of the location of the mid depth of the complete diagonal crack within the shear span . This location is specified for all shear span lengths in Fig. B.19.

Others have suggested locations for the critical section in a shear span. The following list shows a reference with the critical location used which agreed with the data from the corresponding tests:

Reference	Critical Section Location X_{cs} $(a \geq 2d)$
B.10	0.5 a
B.17	0.6 a (a - 2d for a > 5d)
B.27	0.5 a
B.30	a - d

All tests involved were with simple beam conditions, and the cracking stress equations used in each case were of a similar basic form to the one used in the criteria.

Within a shear span associated with a simple beam structure with a single concentrated force, the lowest critical cracking stress defined by the criteria is at the point of the load. That section is the most conservative location for the critical section. However, it is not realistic to use this section because the crack has to have some room to form before the load is encountered. Therefore, the critical section is usually chosen some distance from the point of the load. If the crack is assumed to have a projected length equal to d, then (a - d) is the most conservative realistic choice. Test observations indicate that the location is actually closer

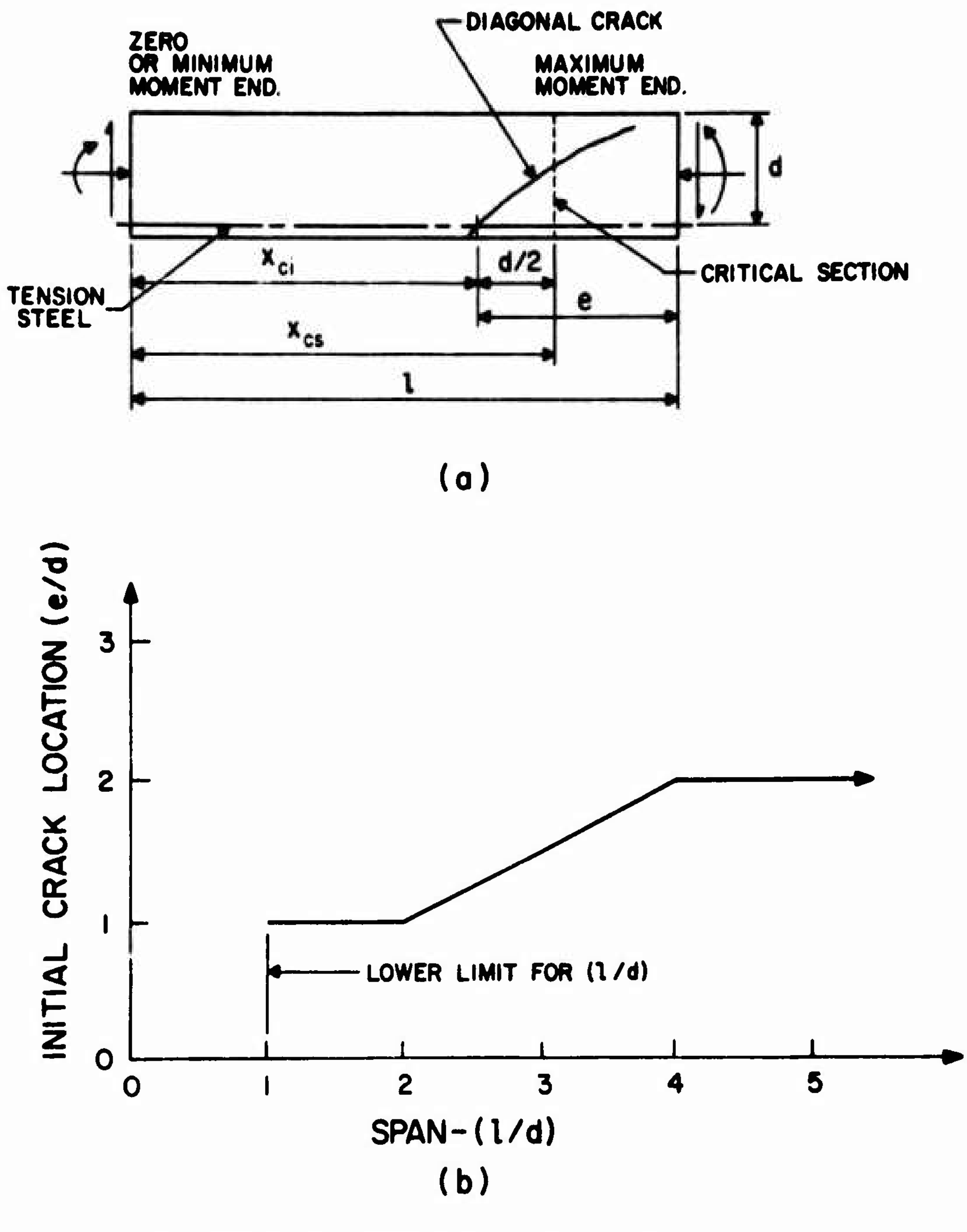


FIG. B. 19: CRITICAL SECTION FOR DIAGONAL CRACKING

to the middle of the span a. The location used with a specific cracking equation must be balanced with the form and conservativeness of the equation itself to produce realistic and conservative results. This is the final decision basis. The location chosen for the criteria is a reasonable balance between the most conservative location and the observed locations. It has also been checked with a variety of test data and compared with the ACI Code equation. These results are shown in Fig. B.20.

A summary of the locations for critical sections associated with each failure criteria is shown below:

	Failure Criteria	Critical Sections for an element
1.	Flexural failure:	End sections, at the nodal points, or at the face of an intersecting member.
2.	Shear-flexure failure:	Section at the mid-depth of the diagonal crack - defined in Fig. B.19.
3.	Axial force failure:	(same as for flexural failure)

B.4.3 CHECKING PROCEDURE

The two general element states, characterized by the internal stress distribution, are represented by element (a - b) and element (b - c) in Fig. B.18. In the first, there is a zero moment condition within the element; the second has no such condition.

In the first element (a - b), the flexural and axial force checks can be made at the critical sections for the element as defined previously. However, the shear check must be made for both shear spans defined; i.e. from the zero moment section to each nodal point (shear spans 1 and 2 in

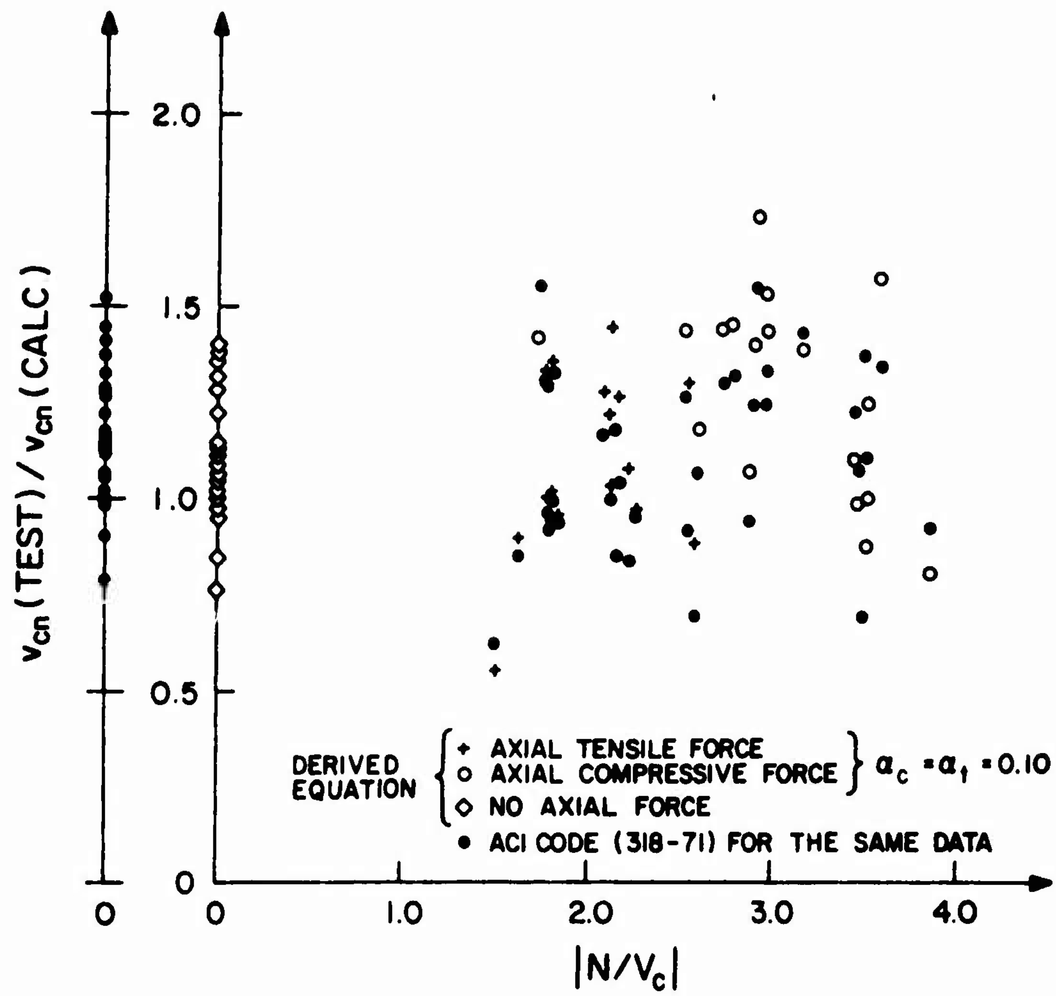


FIG. B. 20: CHECK OF THE DERIVED EQUATION FOR SHEAR-FLEXURE FAILURE (DATA FROM REF. [B. 14]).

the figure). Therefore, the zero moment section must be located. It is defined as the section with a uniform strain distribution.

In the second element state, (b - c), the flexural and axial criteria are applied as before, while the shear criteria can be applied to the given state directly. The application of the shear criteria to this element is equivalent to applying these expressions to a shear span defined by the length (g - c) in Fig. B.18. The length (g - c) is referred to as the equivalent shear span for element (b - c), and it has the approximate value given by:

M_b = bending moment at b

V_{bc} = shear in (b - c)

L = length

$$L_{gc} = \frac{M_b}{V_{bc}} + L_{bc}$$

When a single member is composed of several elements, the question arises concerning the adequacy of an independent check of each element to represent the behavior of the whole member. In other words, is it possible that the total member might fail if it is modeled as a single element, whereas the independent checking of component elements might not detect failure? It has been demonstrated by physical testing of restrained beams that the most likely region for cracking is in the span with the largest shear force, such as element (c - d); (reference B.26).

To verify that the criteria expressions would be consistent with this behavior, an idealized analysis was performed using a member composed of two elements subjected to the possible combinations of bending moment in each span, e.g. the two spans (a - b) and (b - c). The independent checks produced realistic failure predictions. The results showed that the cracking

is less likely in (b - c) and more likely in an adjacent span with larger shear force (a - b) as the moments at b and c approach equality. It was also shown that if the bending moments M_a, M_b and M_c define a linear function with x, and $M_a = 0$, then the failure of element (b - c) would be identical to the failure of the total member (a - c) provided the same critical section is used, and provided both spans have identical properties. Therefore, checking each element of a member independently will accurately reflect the physical behavior of the total member; i.e., the most likely failure region will be detected first.

The general procedure used in checking for failure is indicated in the flow sequence shown in Fig. B.21, where,

critical strain values	= strain values at the extremities of the end sections of an element;
section of zero moment	= section with a uniform strain state;
equivalent shear span	= distance from the zero moment section (outside of the element length) to the section of maximum moment.

Flexural failure checks (B.3.1) are required at both ends rather than just at the end with the maximum moment since an element may be reinforced unsymmetrically with respect to number, diameter, and location of longitudinal bars. By the same reasoning, the axial force failure checks (B.3.3) are also made at both ends of an element.

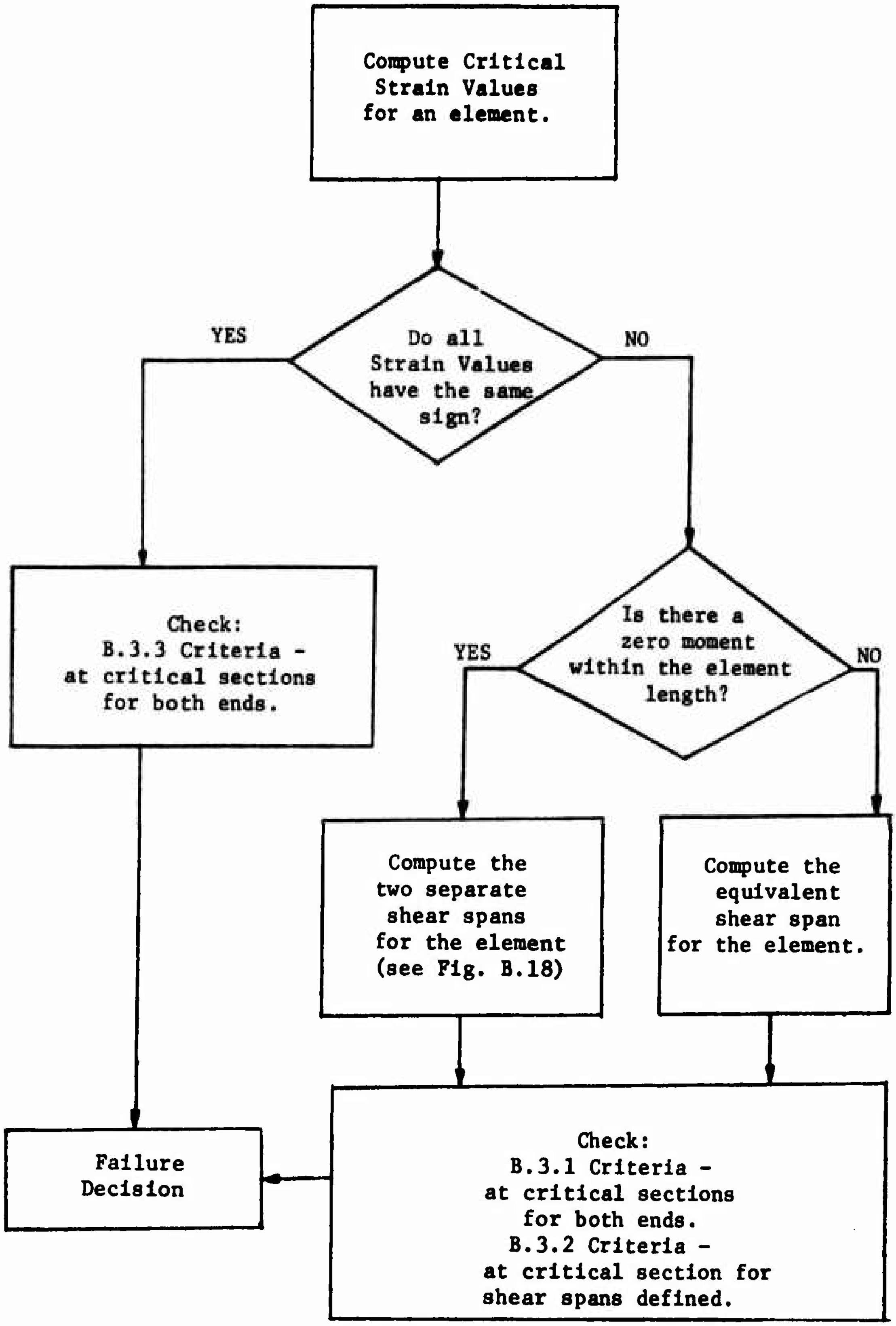


Fig. B.21: General Sequence for Implementation of the Failure Criteria

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NOTATION

b = gross cross section width

d = effective depth of longitudinal reinforcing bars

h = total depth of cross section

s = spacing of web reinforcing bars

l = length of shear span for checking diagonal cracking failure

a = shear span measured from zero moment to a concentrated load

D = reinforcing bar diameter

k = radius of gyration of reinforcing bar

x_{ci} = distance to diagonal crack intersection with tension steel

x_{cs} = distance to critical section from minimum moment

A_s = area of reinforcing bars as a group

A_b = area of a single reinforcing bar

A_v = A_b x number of legs = effective area of web reinforcement bars (Closed stirrups: $A_v = 2 \times A_b$)

p = A_s/bd = Longitudinal steel percentage

r = A_v/bs = dimensionless measure of web reinforcement area

n = d/s = number of bars crossing a crack (at 45°)

ε_{20u} = unconfined concrete strain at 0.20 f'_c beyond the ultimate strain

ε_{50u} = unconfined concrete strain at 0.50 f'_c beyond the ultimate strain

ε_{f1} = strain in concrete at crushing in compression

ε^*_{f1} = strain in concrete at crushing with modification coefficient

ε_c = strain in concrete in compression

ε_{f2} = strain in longitudinal steel at fracture in tension

ε_s = strain in longitudinal steel in tension

ε_v = strain in web reinforcing bar

f'_c = ultimate cylinder strength of concrete

$\sqrt{f'_c}$ = a measure of the tensile strength of concrete

f_s = stress in longitudinal reinforcing bar

f_{cr} = compressive stress in longitudinal reinforcing bar at critical condition

f_v = stress in web reinforcing bar

f_{vy} = stress in web reinforcing bar at yield in tension

v = V/bd = nominal measure of shear stress

v_c = V_c/bd = nominal measure of cracking shear stress without axial force

v_{cn} = V_{cn}/bd = nominal measure of cracking shear stress with axial force (tension or compression)

E_t = tangent modulus for steel stress-strain response

$V(x)$ = V = shear force at section x

$V_c(x)$ = V_c = cracking shear force at section x without axial force

$V_{cn}(x)$ = V_{cn} = cracking shear force at section x with axial force

N = axial force in an element (tension or compression)

P = measure of applied load

P_c = measure of applied load at diagonal cracking

$M(x) = M$ = bending moment at section x

$F(x) = V(x)/bd\sqrt{f'_c}$

$F_1(x) = V_c(x)/bd\sqrt{f'_c}$

$$F_2(x) = 1000.\ \frac{pd}{\sqrt{f'_c}} \left| \frac{V(x)}{M(x)} \right|$$

$F_3(x) = V_{cn}(x)/bd\sqrt{f'_c}$

$F_3^*(x) = V_{cn}(x)/bd\sqrt{f'_c}$ = measure with modification coefficient

$F_4 = N/bd\sqrt{f'_c}$ = nondimensional axial force

C_1, C_2, C_3, C_4, C_5 = modification coefficients (range of values defined with criteria)

α = coefficient for axial force effect on shear-flexure failure

$\alpha_t = 0.025(4. - C_4)$

$\alpha_c = 0.050(2. + C_4)$

APPENDIX C

BIBLIOGRAPHY

This bibliography contains the results of an extensive literature search. The purpose is to document the important sources of information related to the development of a mathematical model to study the complete behavior of a reinforced concrete skeletal structure up to the state of collapse. The literature associated with the total scope of work is so extensive that only those sources related to the development of the basic model are included.

The organization of the entries is by basic subject division, as shown below.

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